

Finite Automata

Part Three

Designing NFAs

Designing NFAs

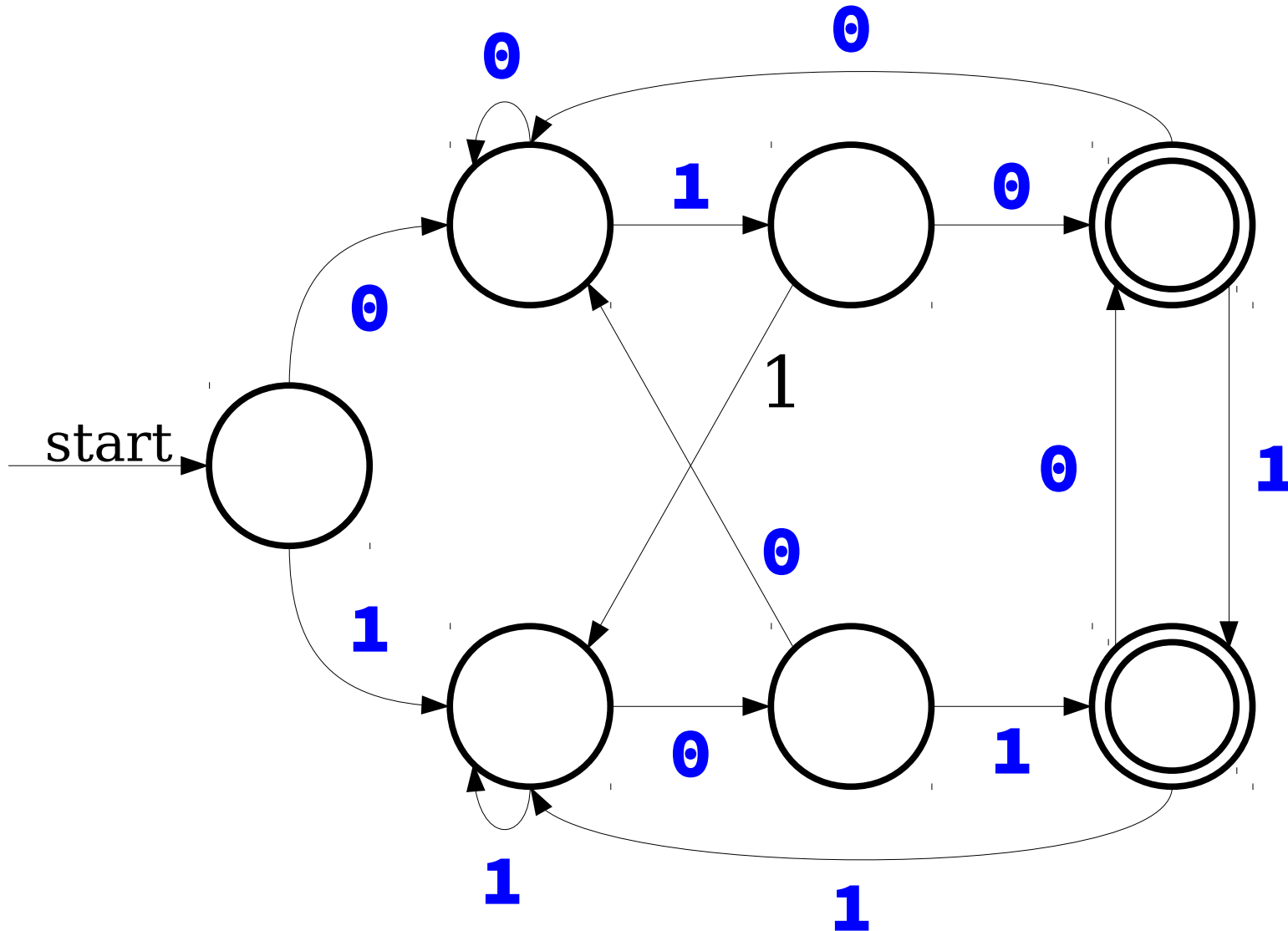
- When designing NFAs, *embrace the nondeterminism!*
- Good model: ***Guess-and-check***:
 - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
 - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

Guess-and-Check

$$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$$

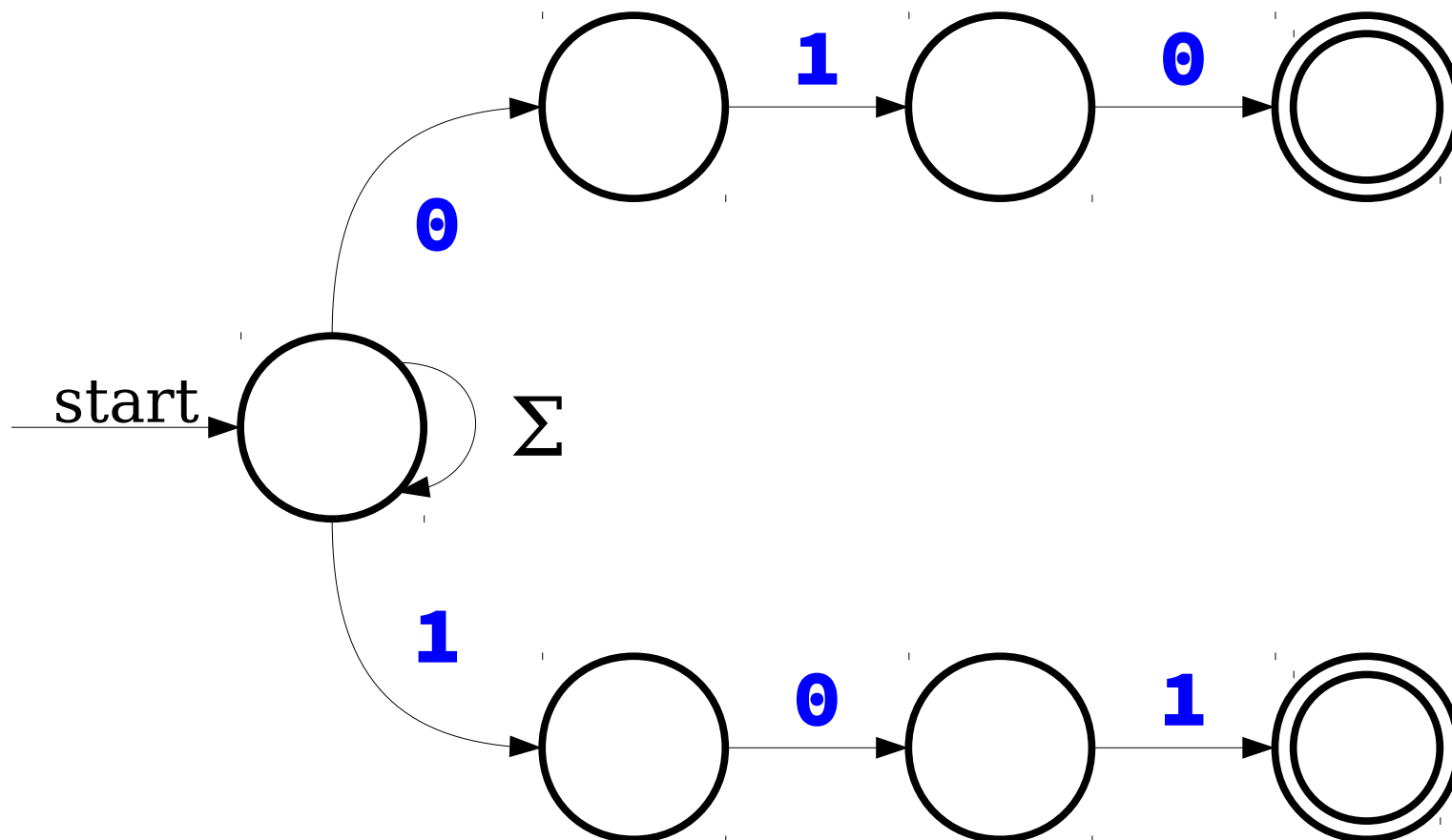
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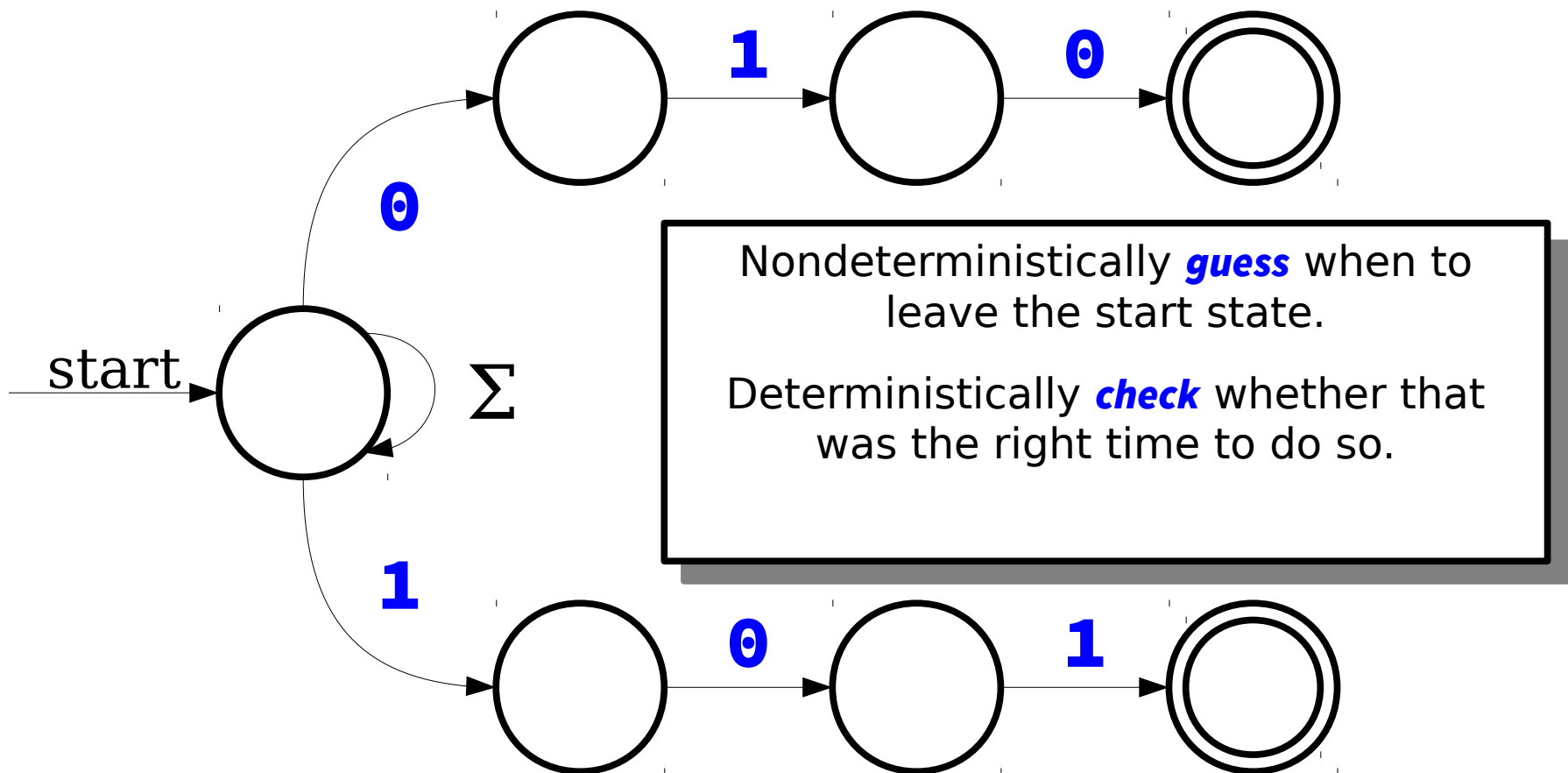
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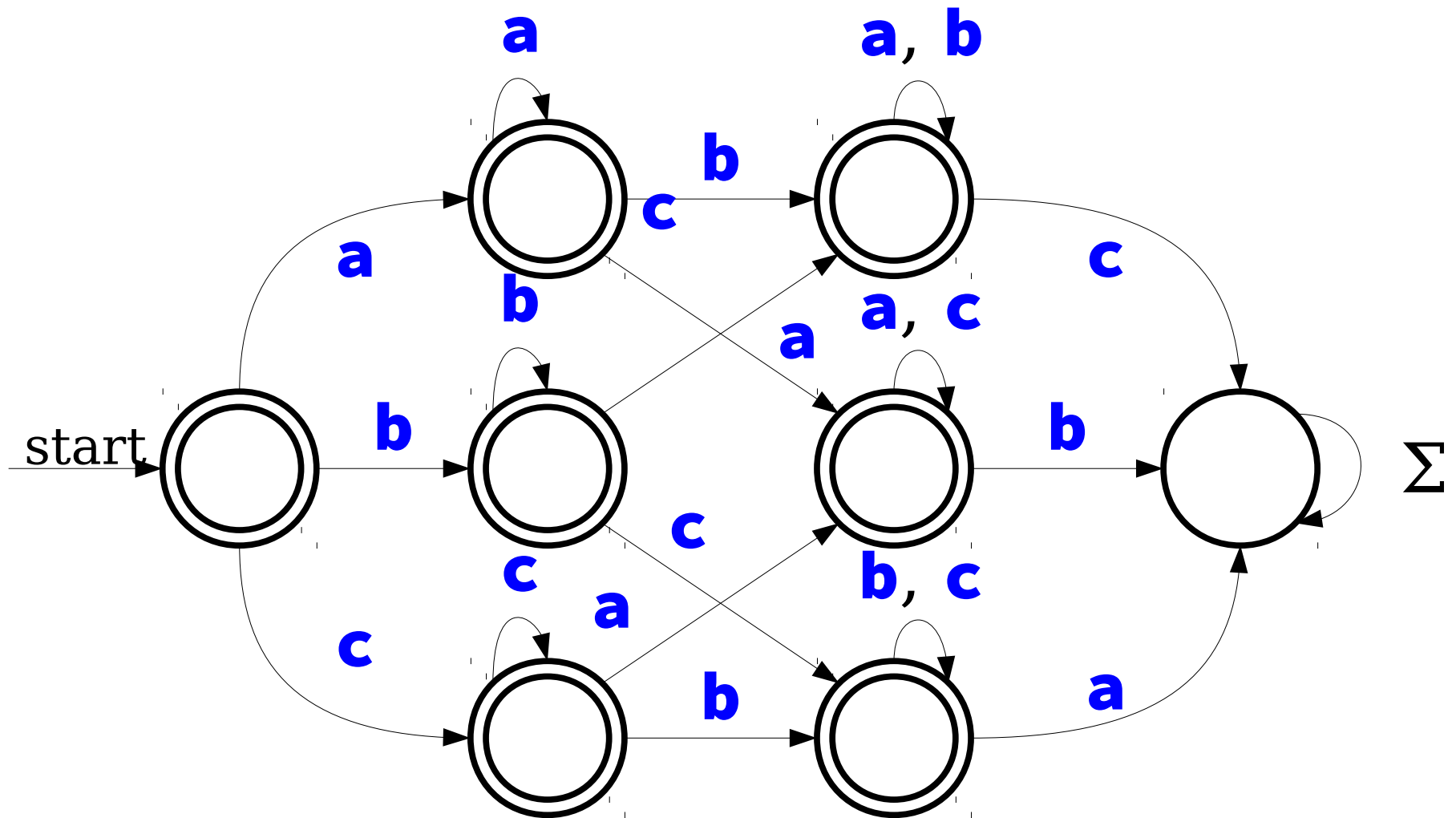


Guess-and-Check

$L = \{ w \in \{a, b, c\}^* \mid \text{at least one of } a, b, \text{ or } c \text{ is not in } w \}$

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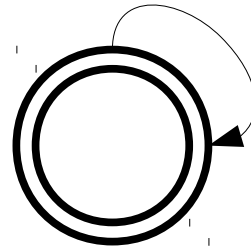


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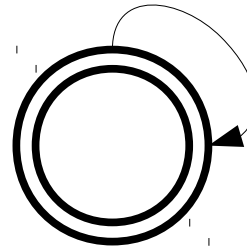
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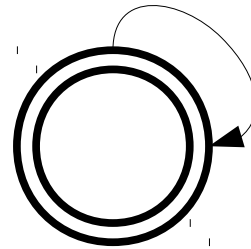
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a, b



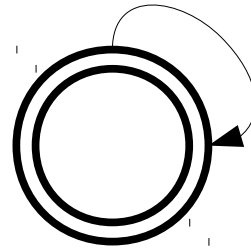
a, c



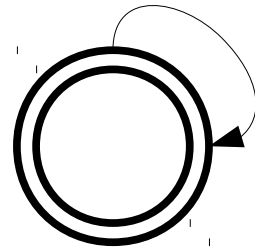
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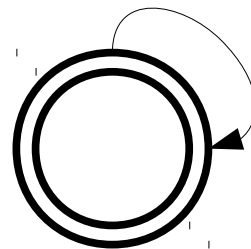
a, b



a, c



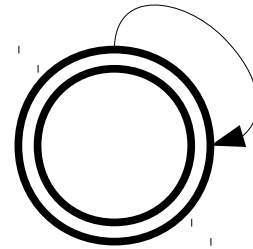
b, c



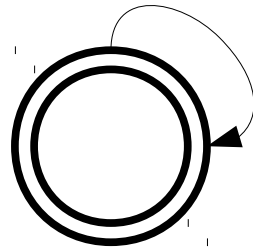
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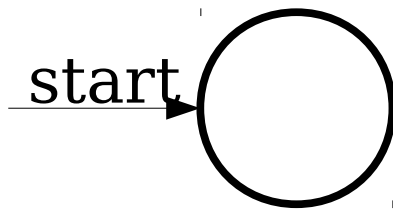
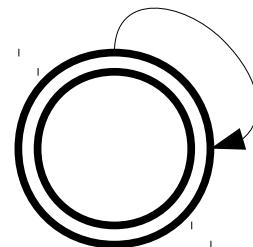
a, b



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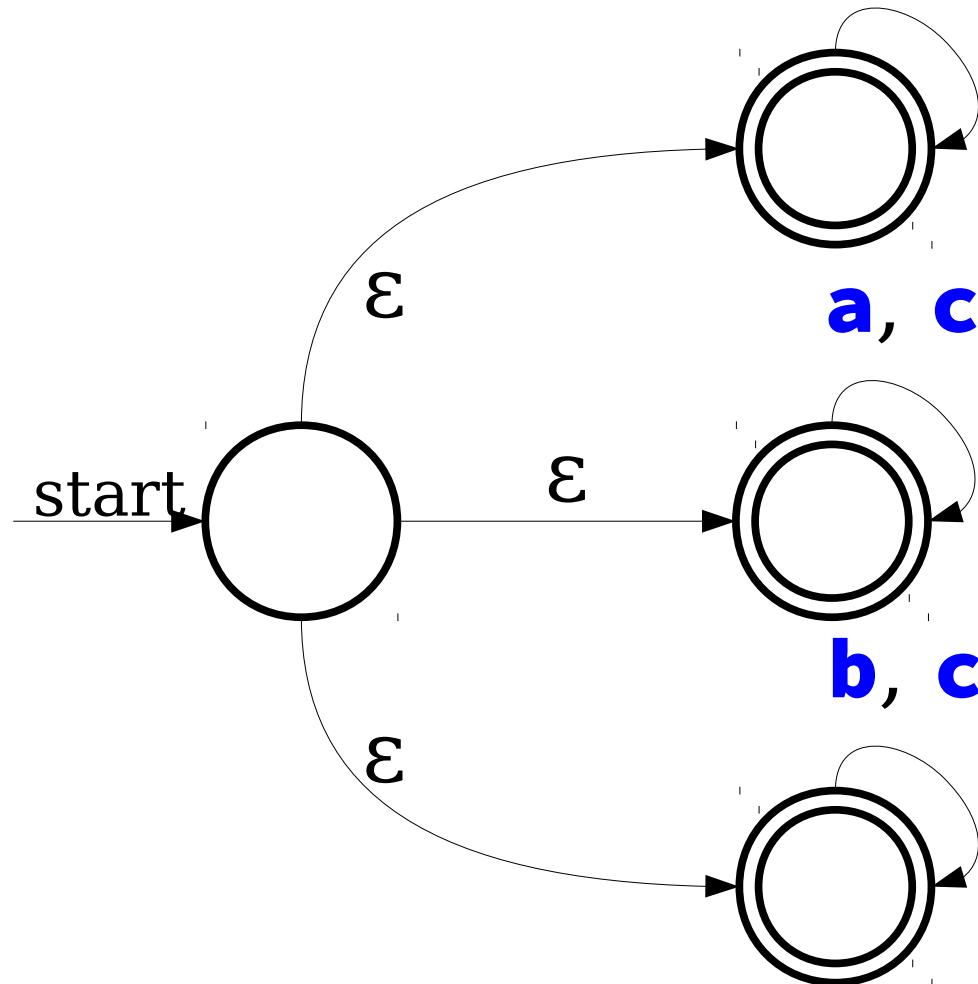


b, c



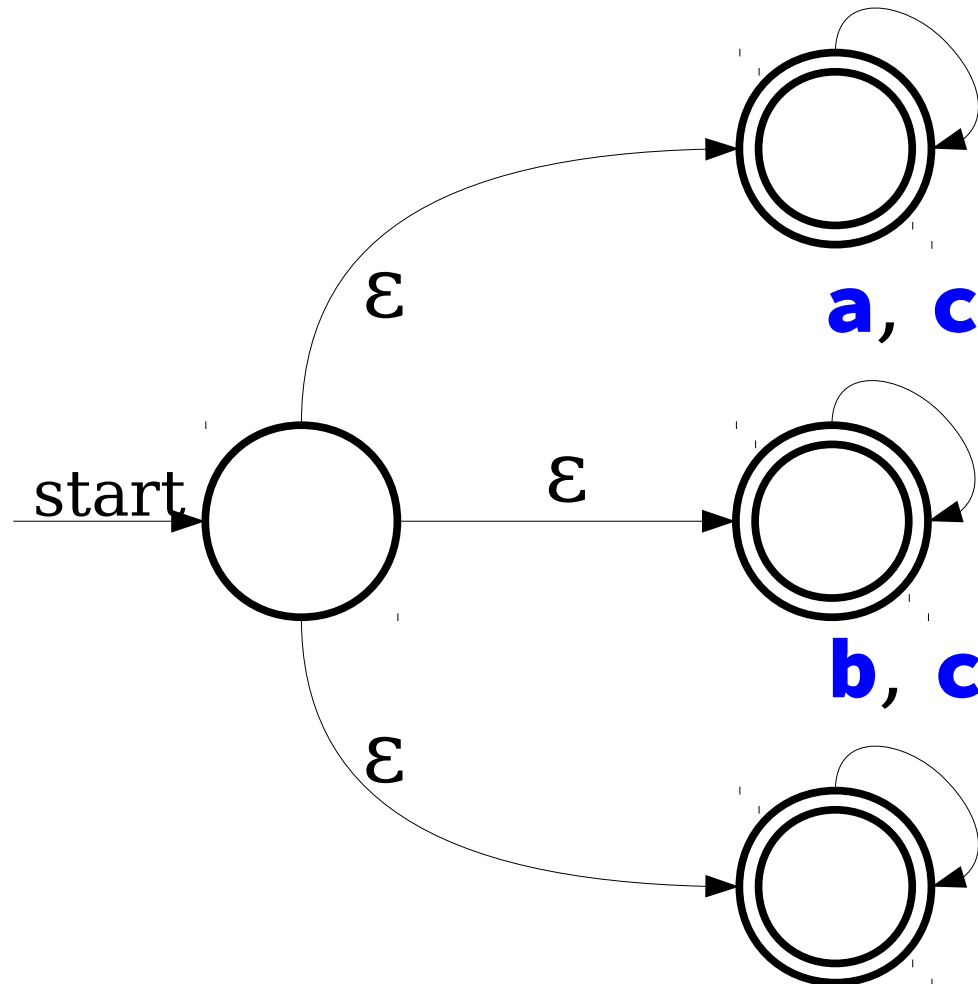
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Nondeterministically **guess** which character is missing.

Deterministically **check** whether that character is indeed missing.

Just how powerful *are* NFAs?

NFAs and DFAs

- Let's just make a wild conjecture that NFAs and DFAs actually have the same power. How could we prove that?
- For all languages L , there exists an NFA that recognizes L iff there exists a DFA that recognizes L .
 - \forall languages L . (\exists DFA D . ($\mathcal{L}(D) = L$) \leftrightarrow \exists NFA N . ($\mathcal{L}(N) = L$))
- **Question:** What is our proof technique for proving biconditionals?

NFAs and DFAs

- Forward direction:
 - \forall languages L . (\exists DFA D . ($\mathcal{L}(D) = L$) \rightarrow \exists NFA N . ($\mathcal{L}(N) = L$))
- True. Why?

NFAs and DFAs

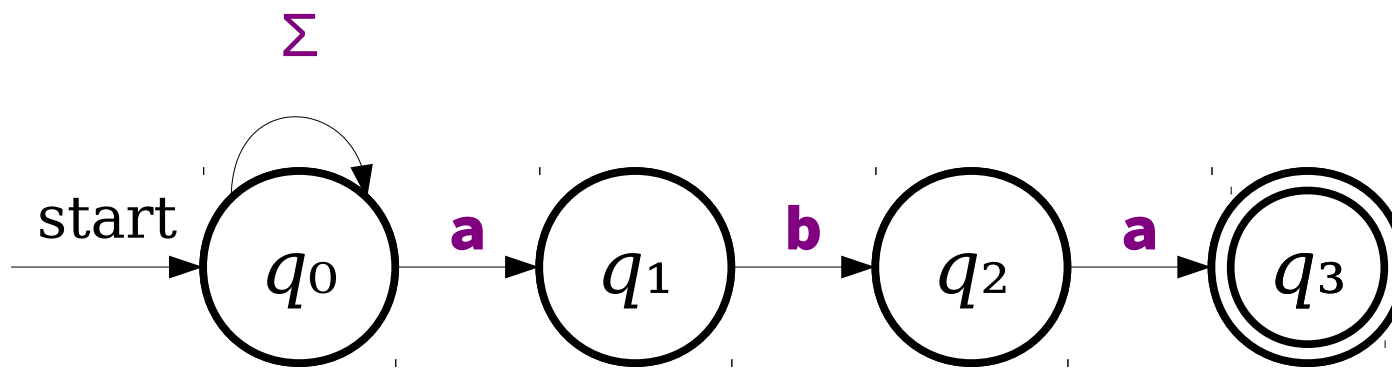
- Forward direction:
 - \forall languages L . (\exists DFA D . ($\mathcal{L}(D) = L$) \rightarrow \exists NFA N . ($\mathcal{L}(N) = L$))
- True. Why?
- Every DFA is already an NFA!
 - NFAs are not *required* to use epsilon edges, 0/2+ transitions, etc.

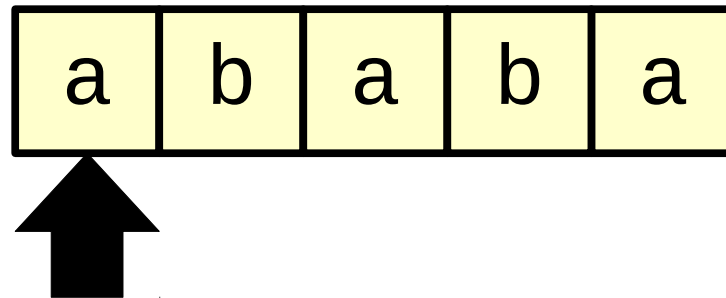
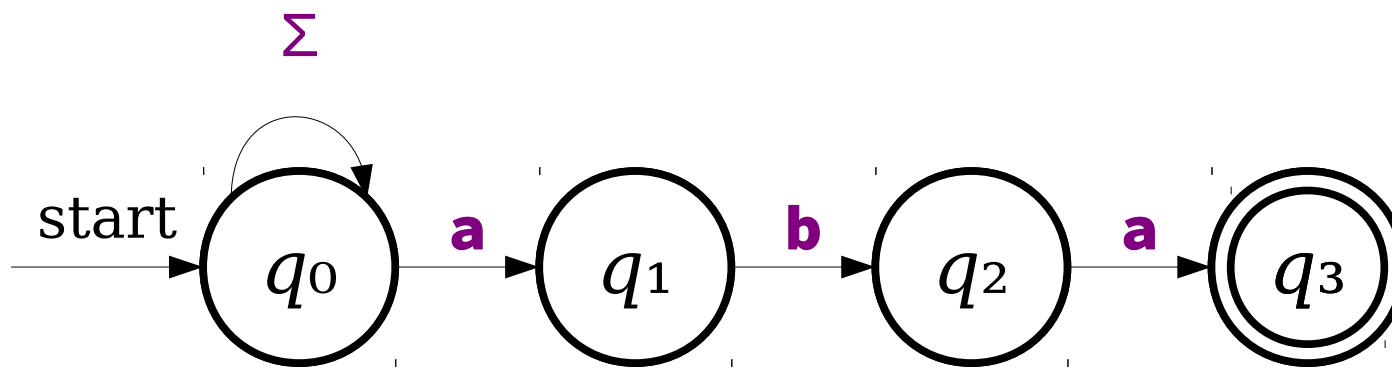
NFAs and DFAs

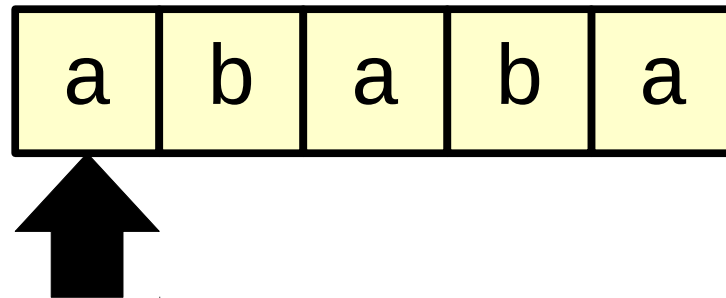
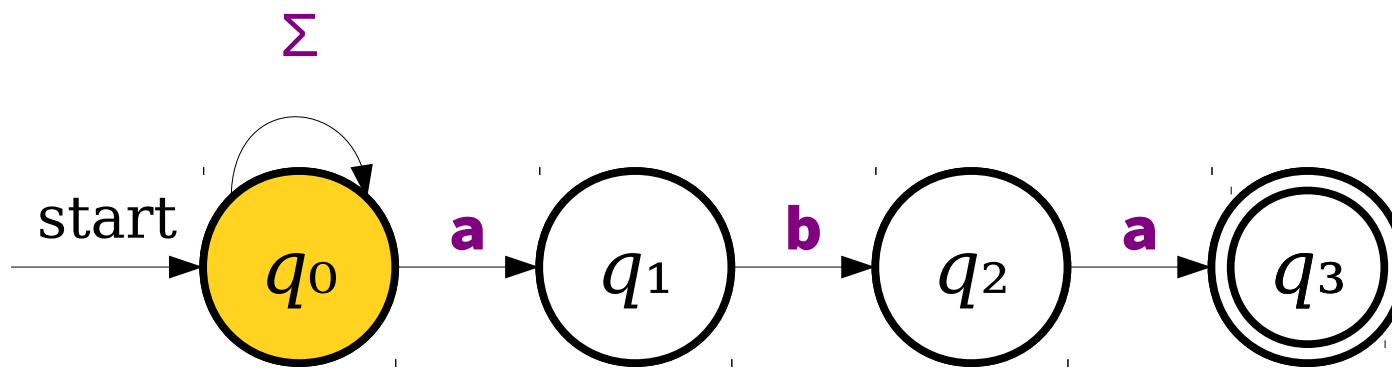
- Forward direction:
 - \forall languages L . (\exists NFA N . ($\mathcal{L}(N) = L$) \rightarrow \exists DFA D . ($\mathcal{L}(D) = L$))
- True. Why?
 - *A little more complicated...*

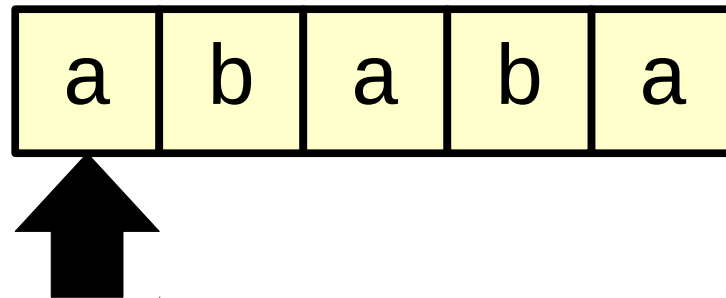
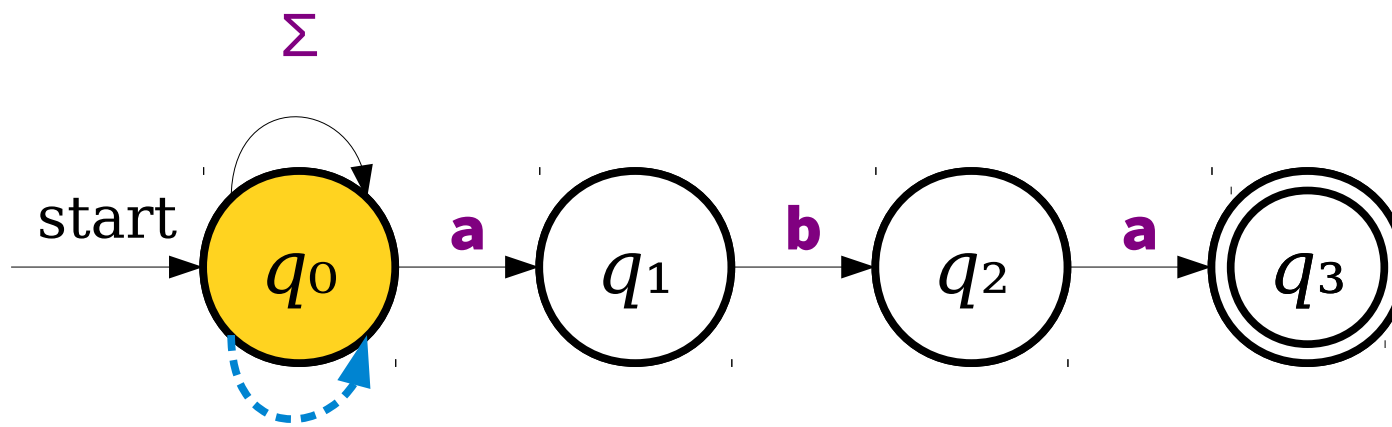
Thought Experiment:

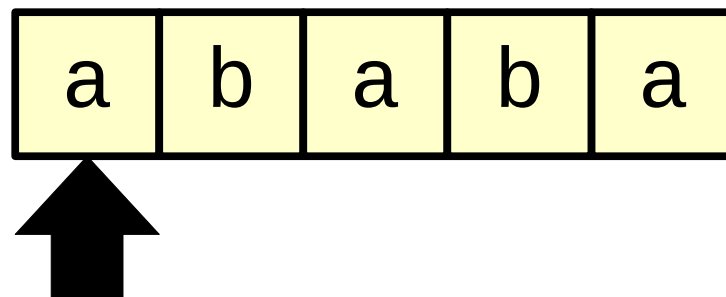
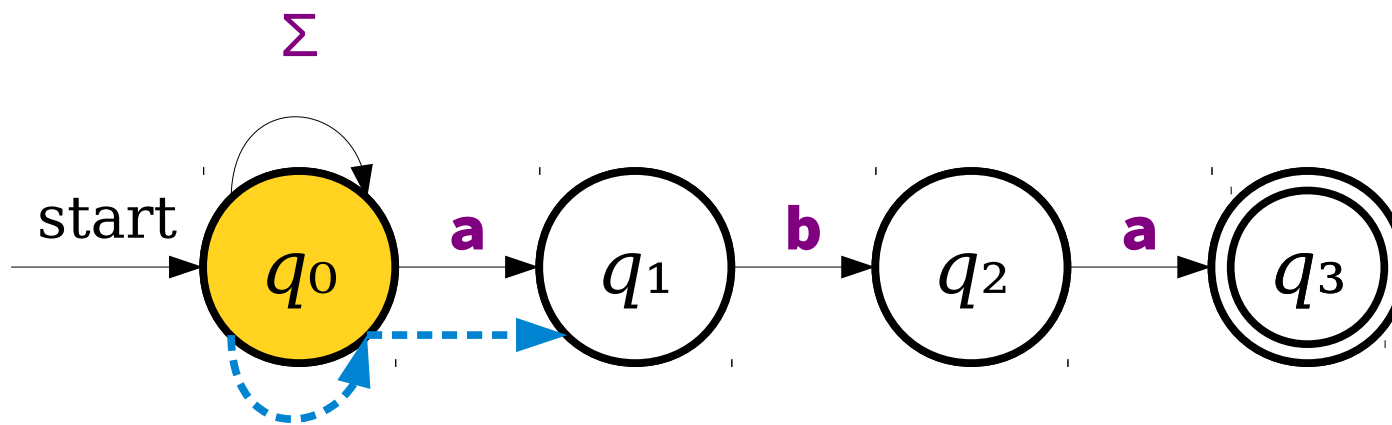
Remember that “Massive Parallelism”
(Voldemort and the horcruxes) thing?

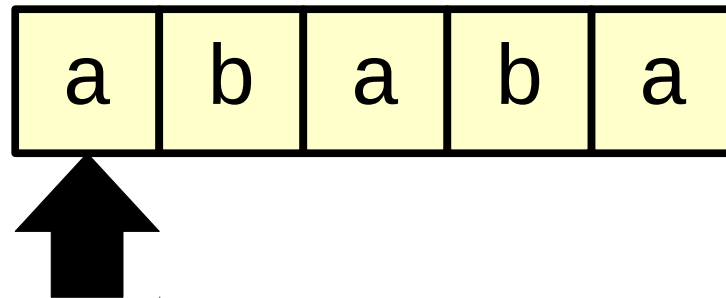
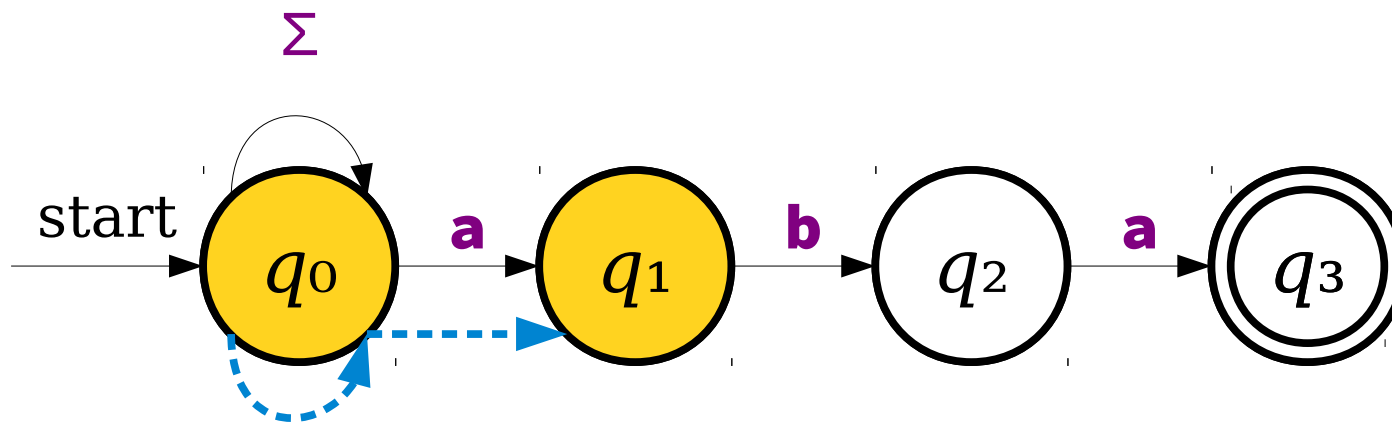


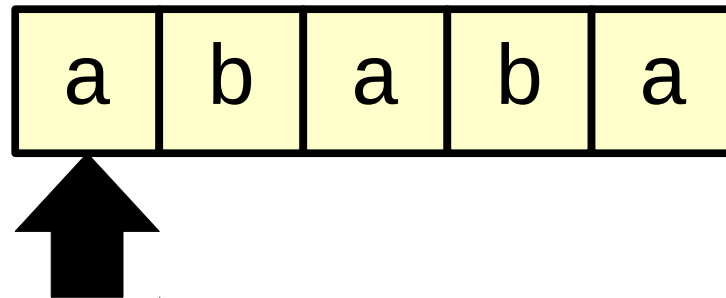
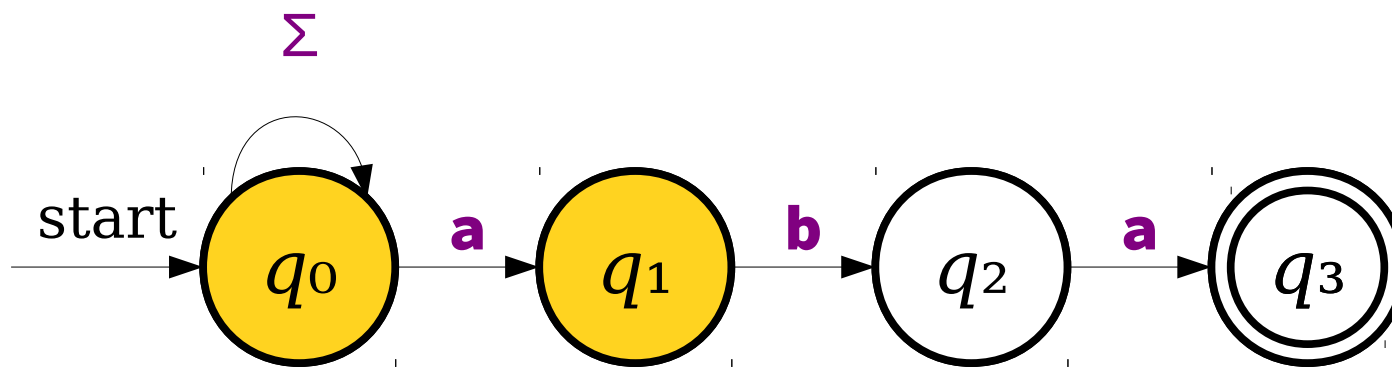


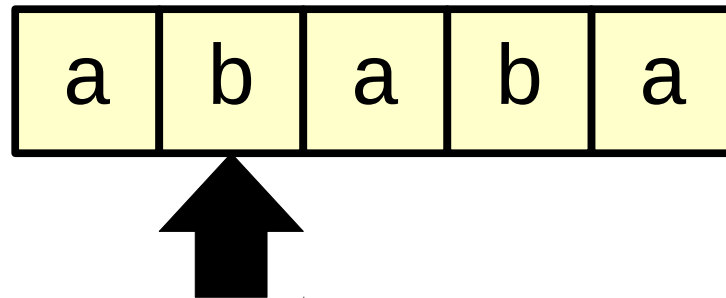
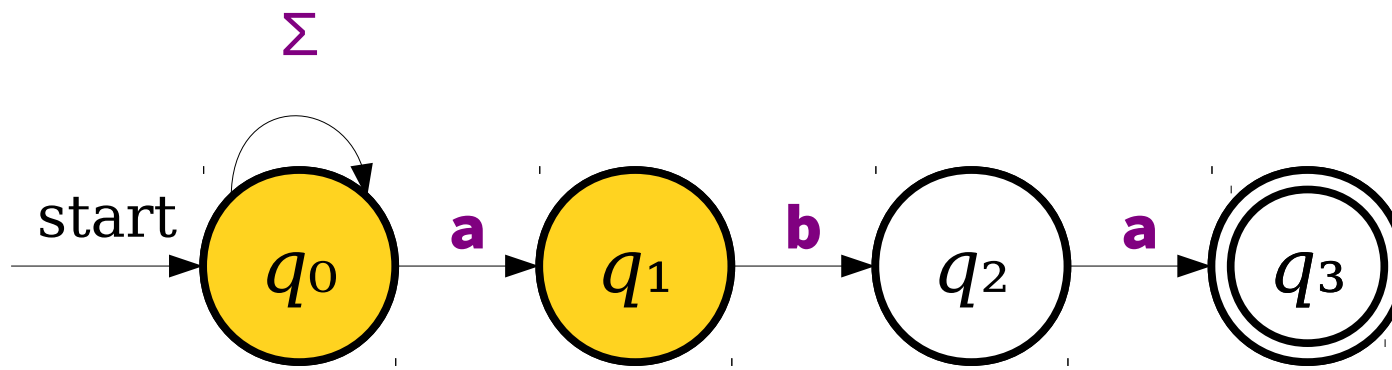


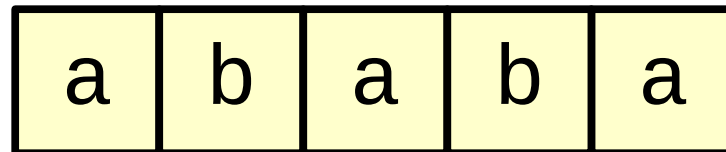
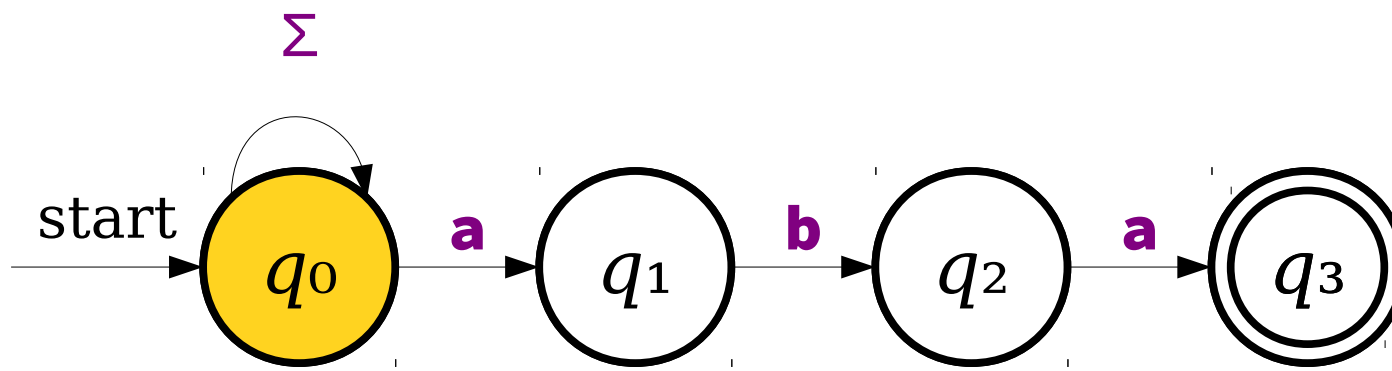


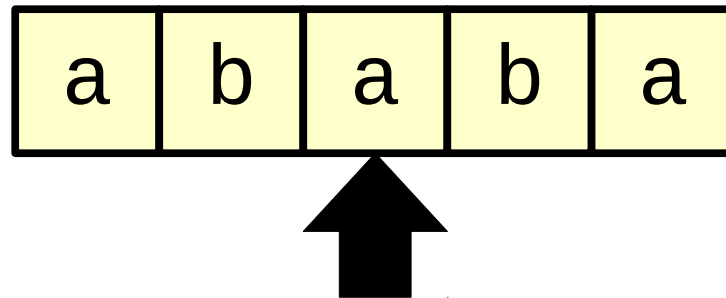
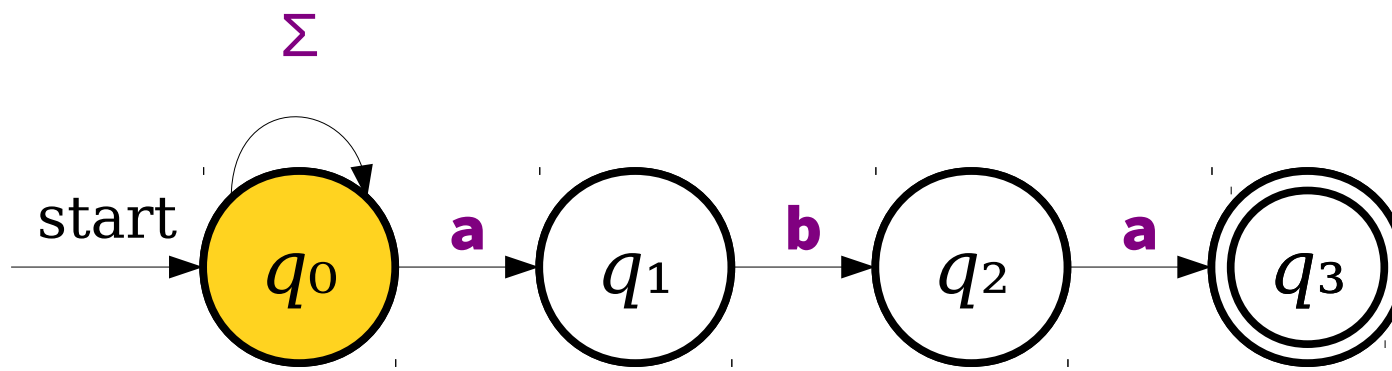


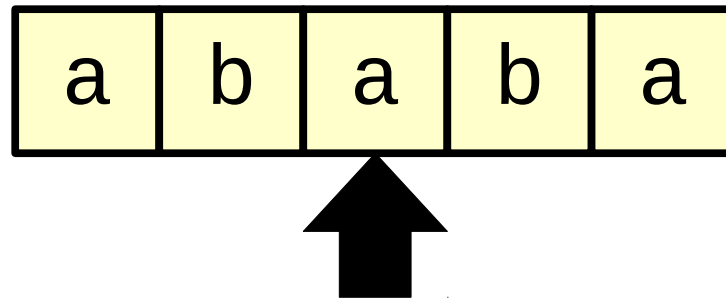
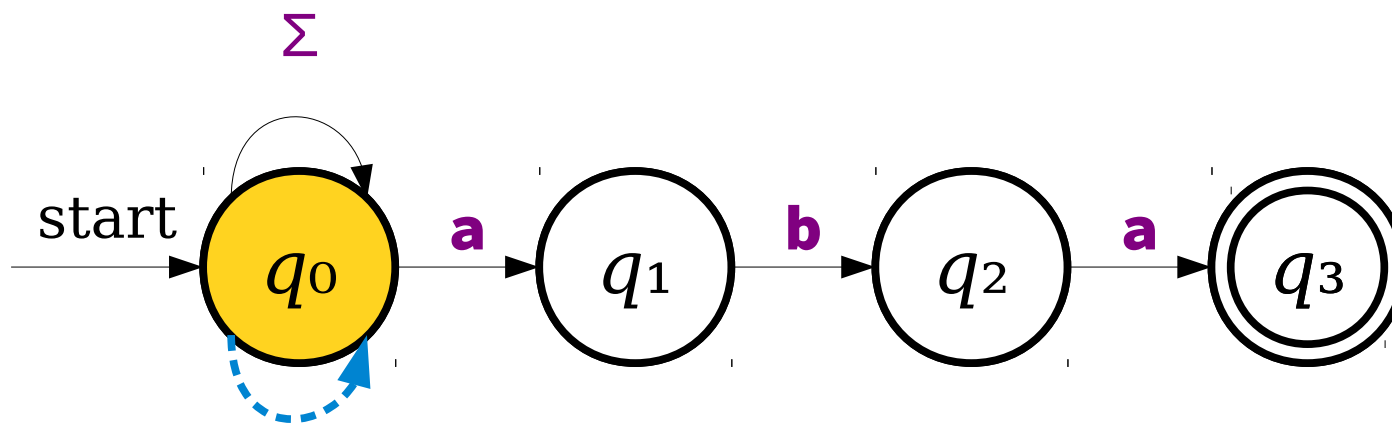


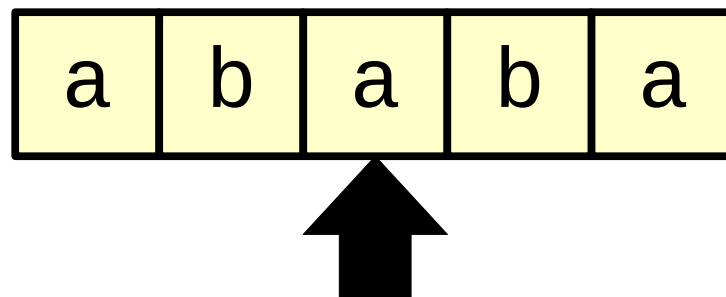
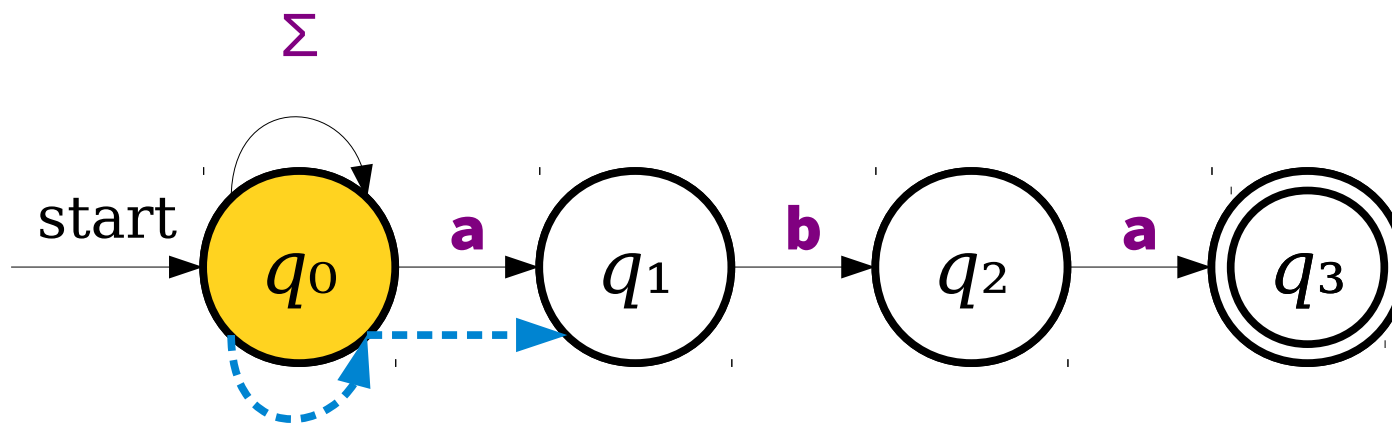


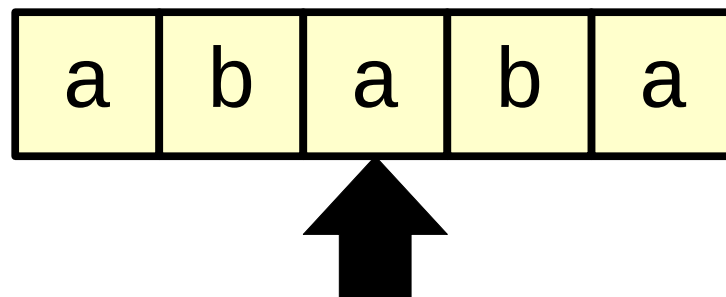
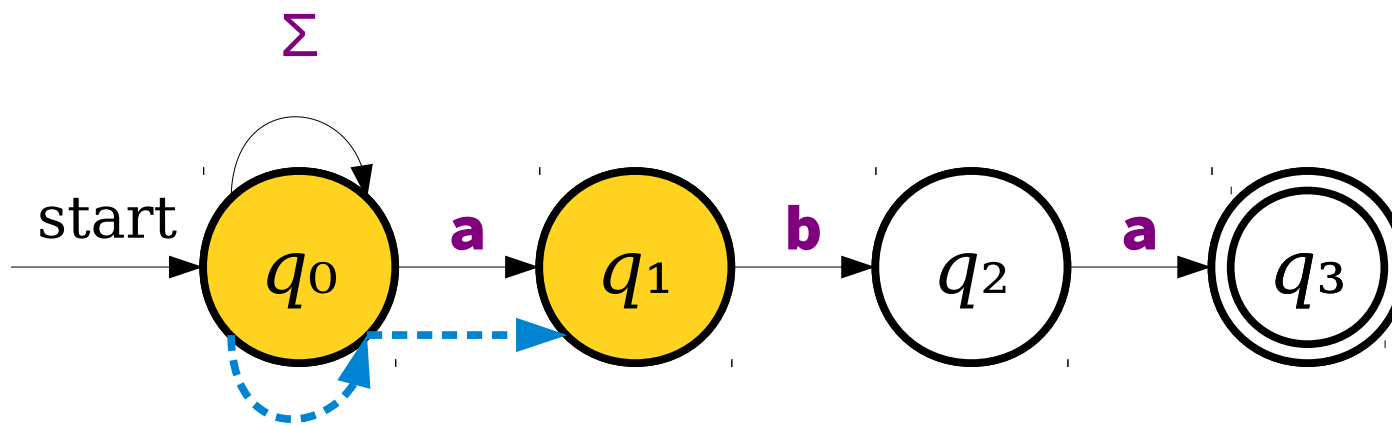


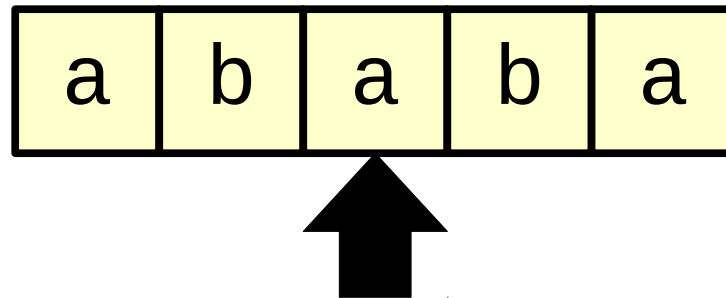
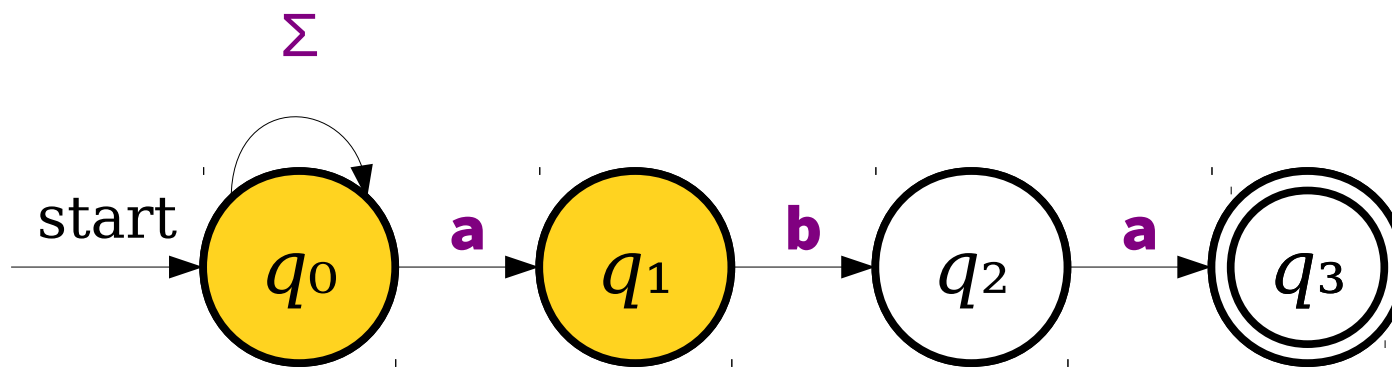


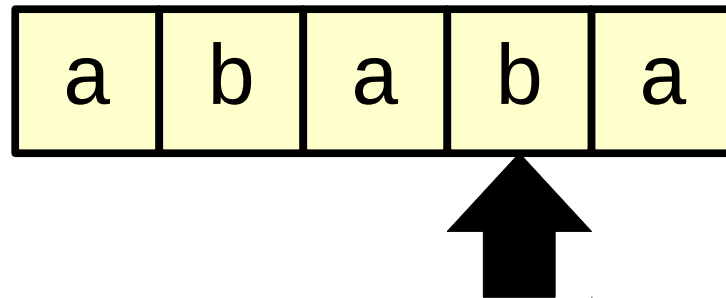
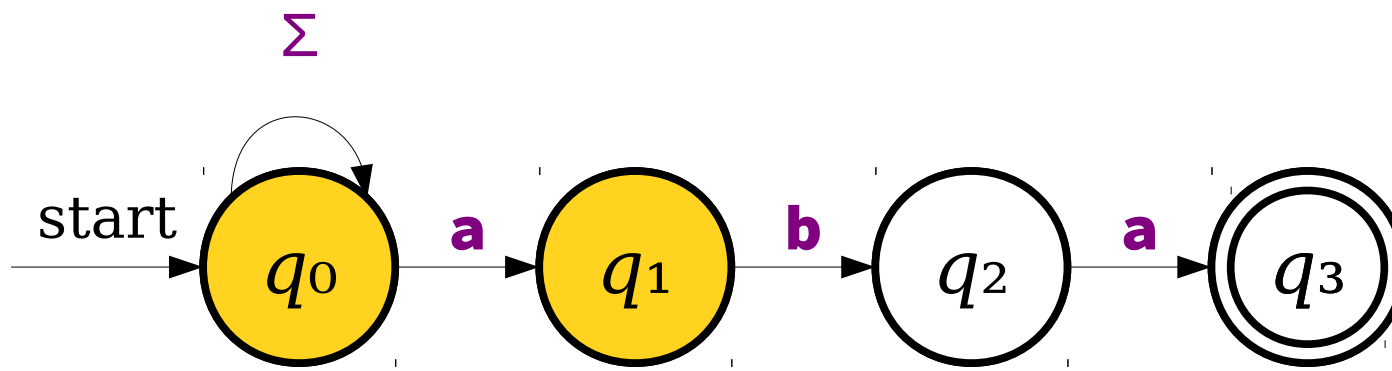


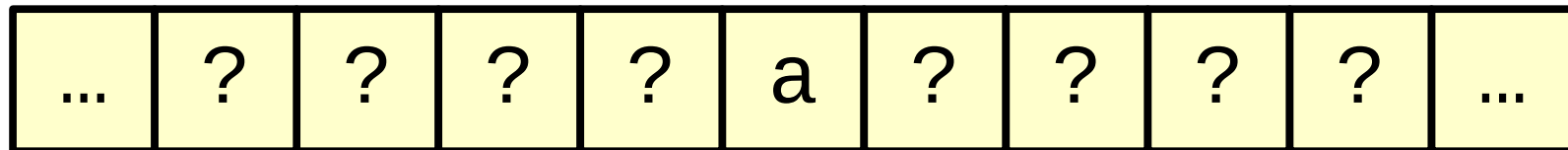
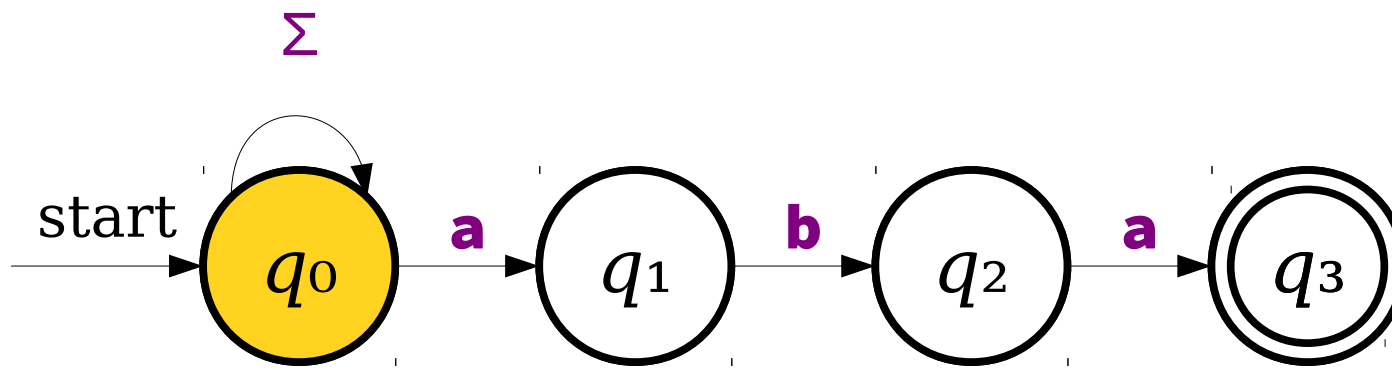


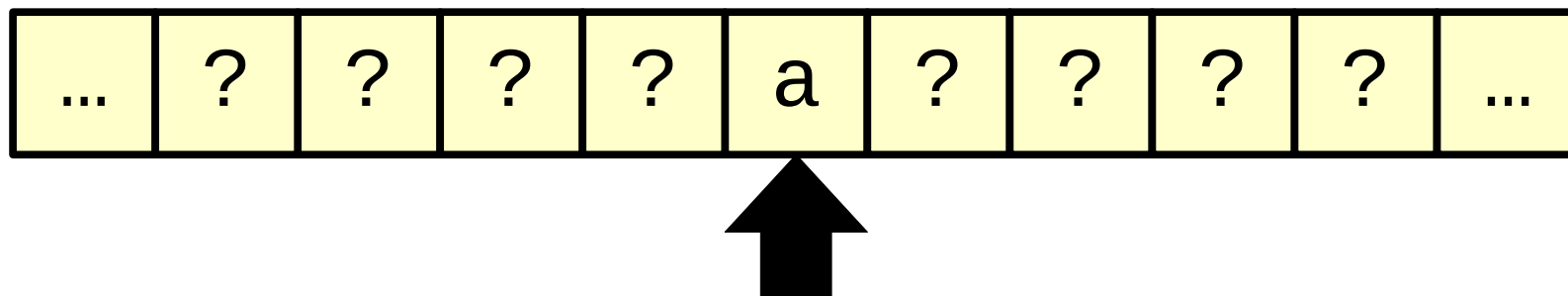
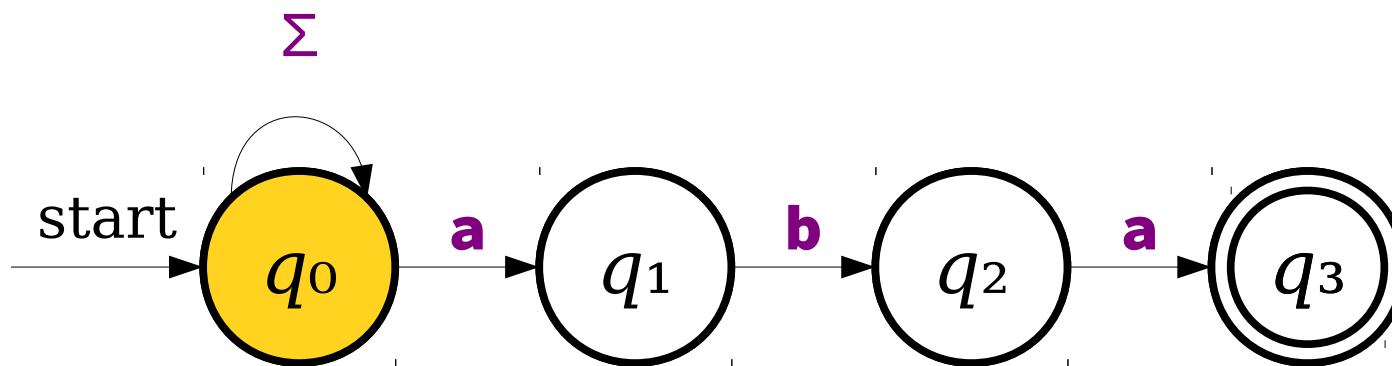


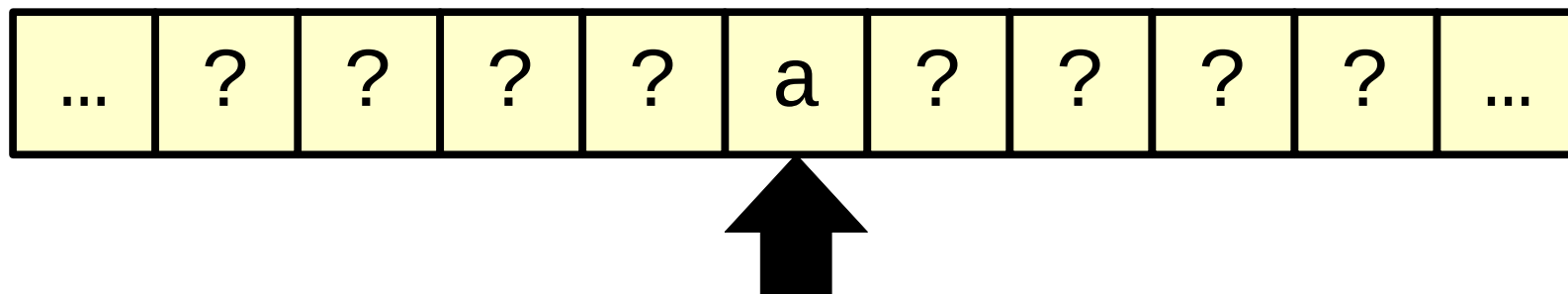
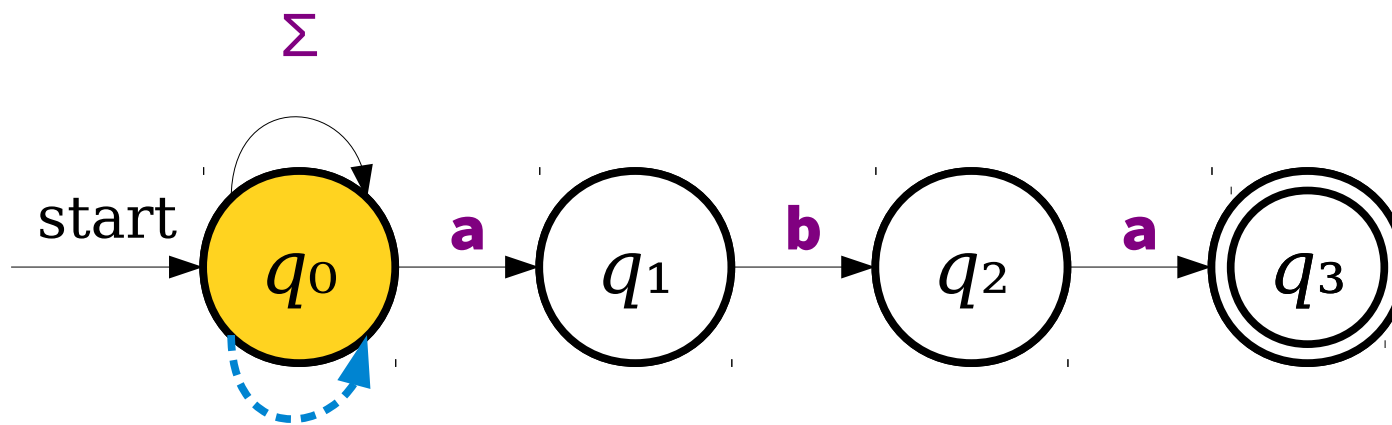


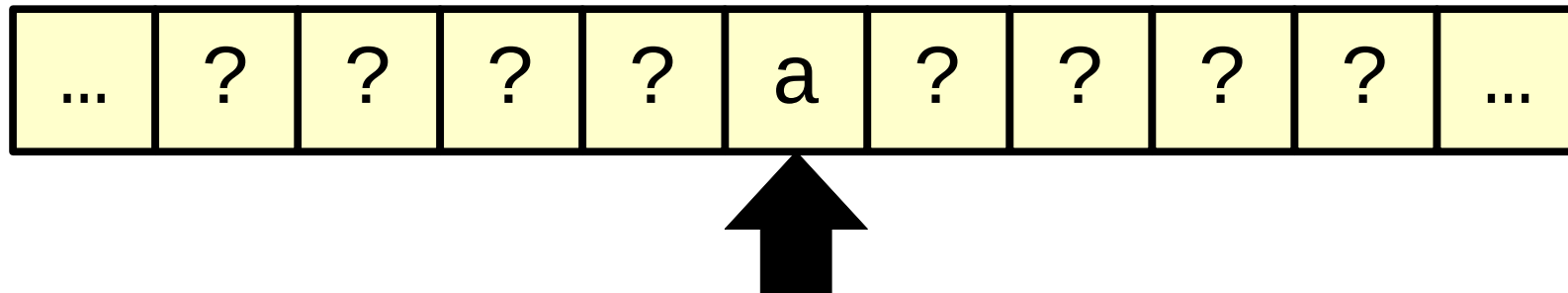
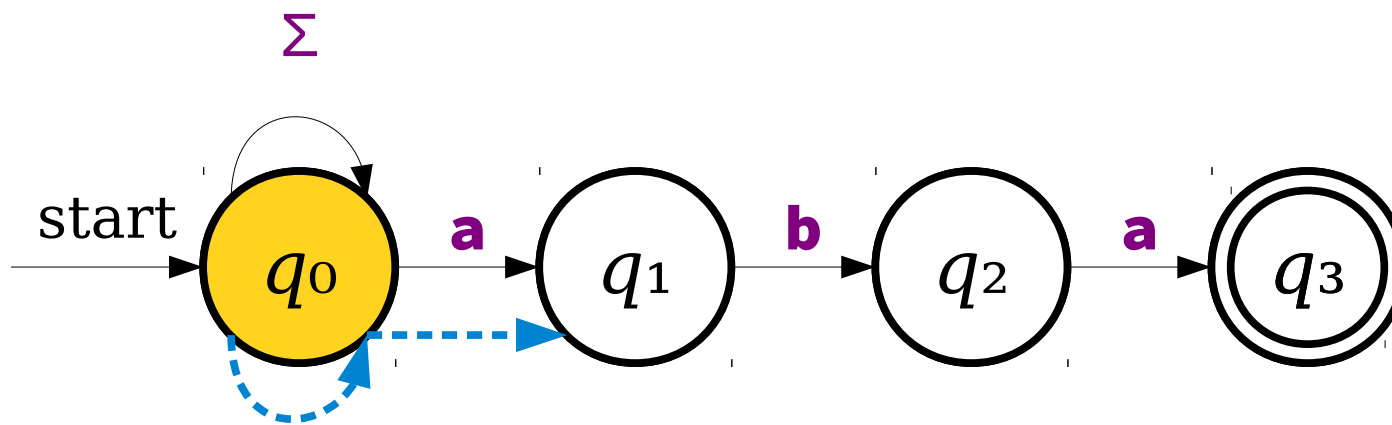


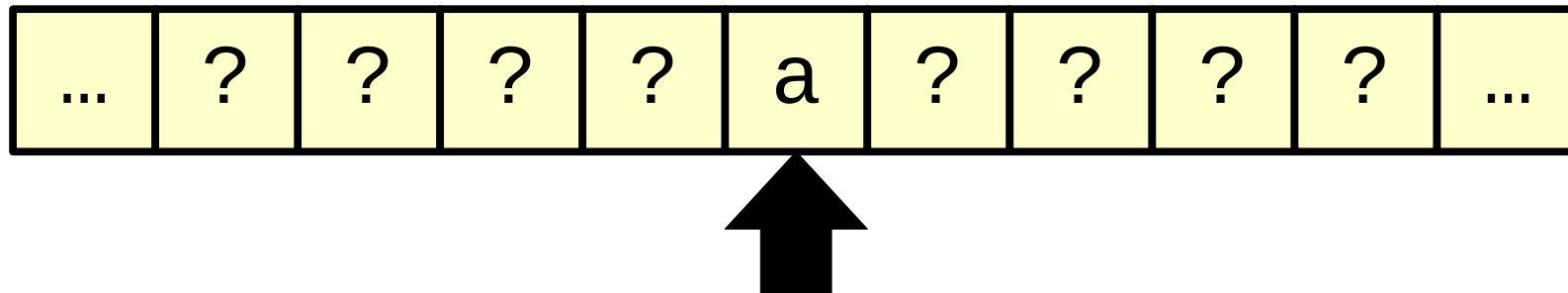
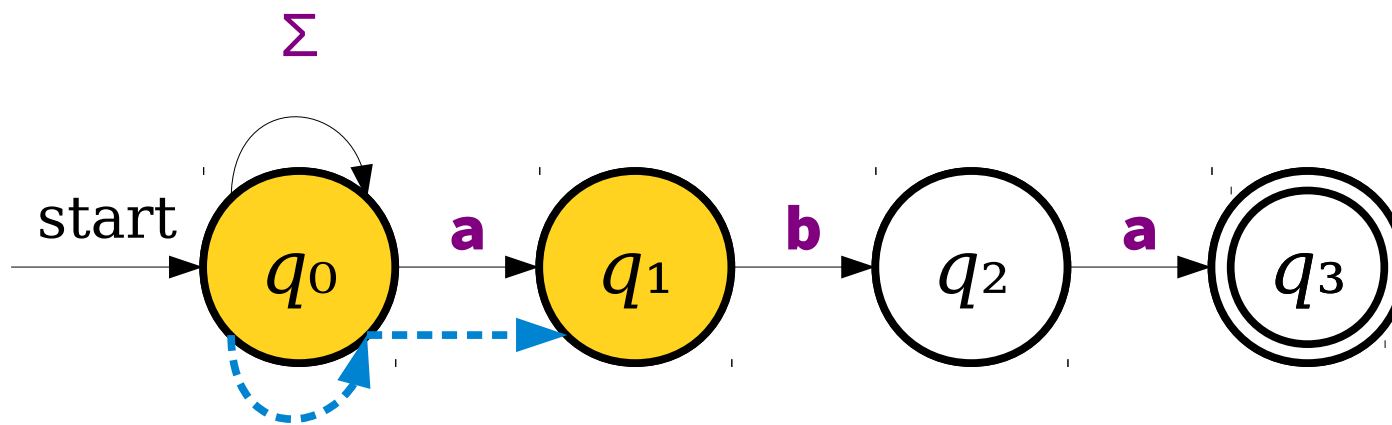


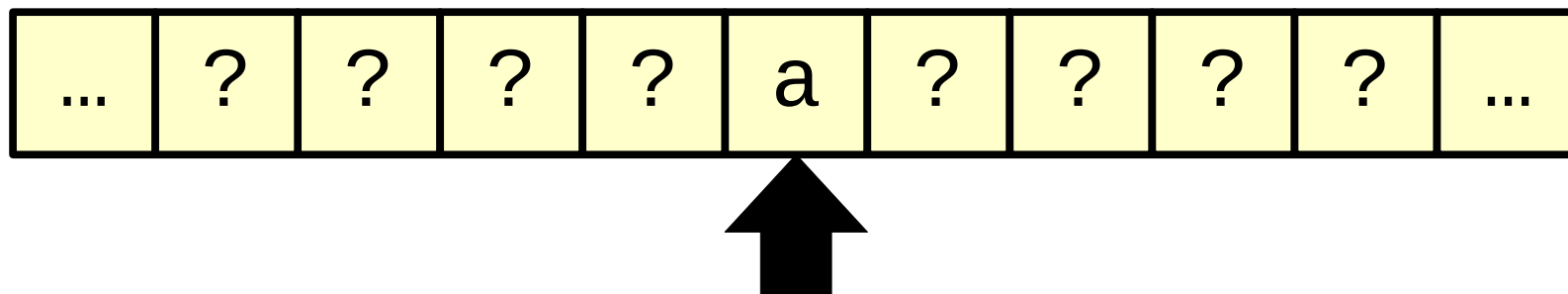
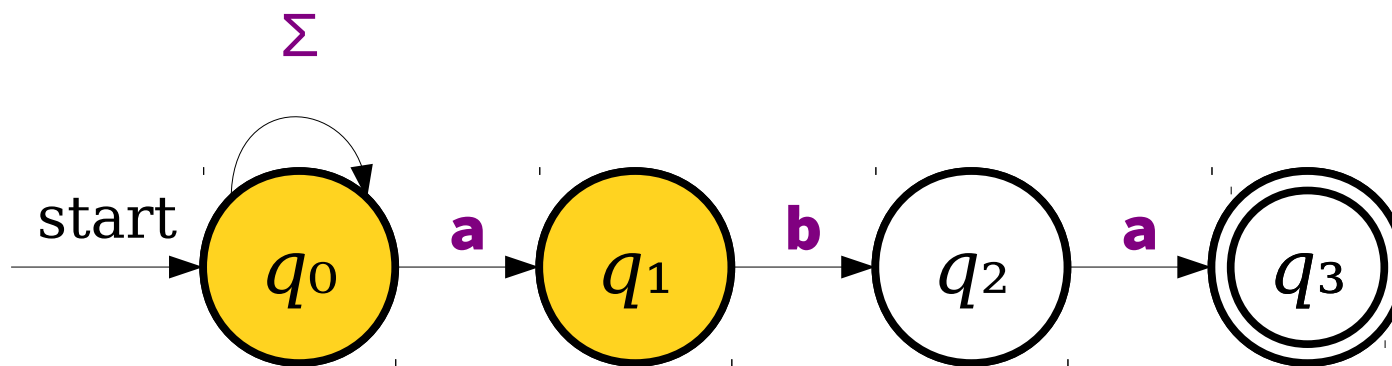


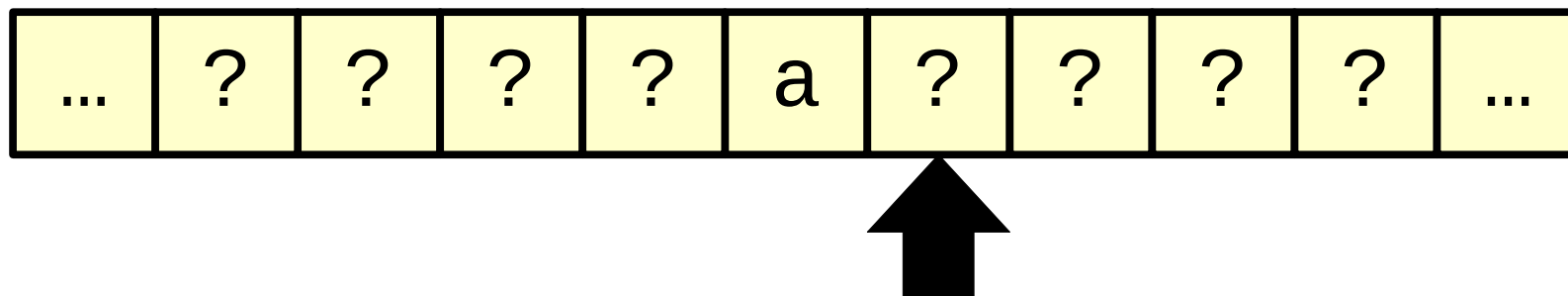
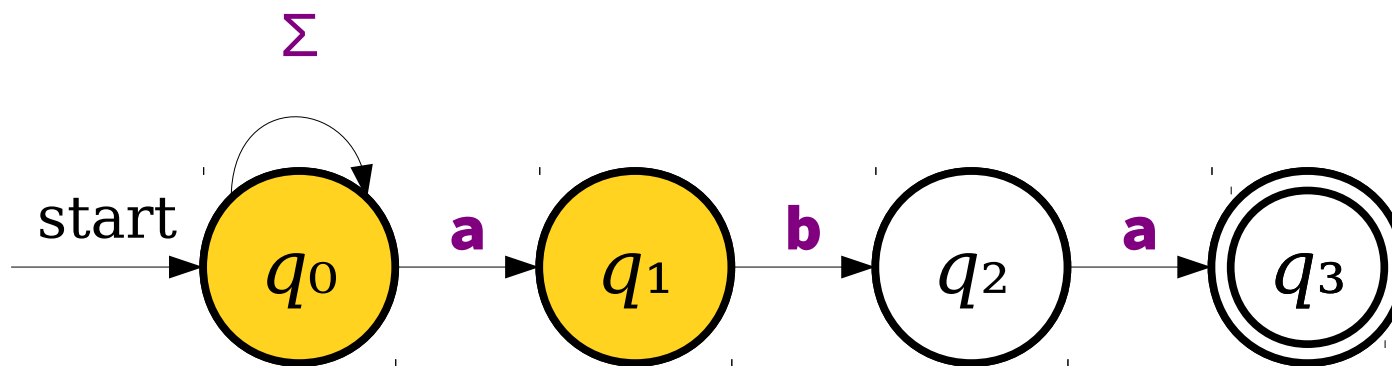


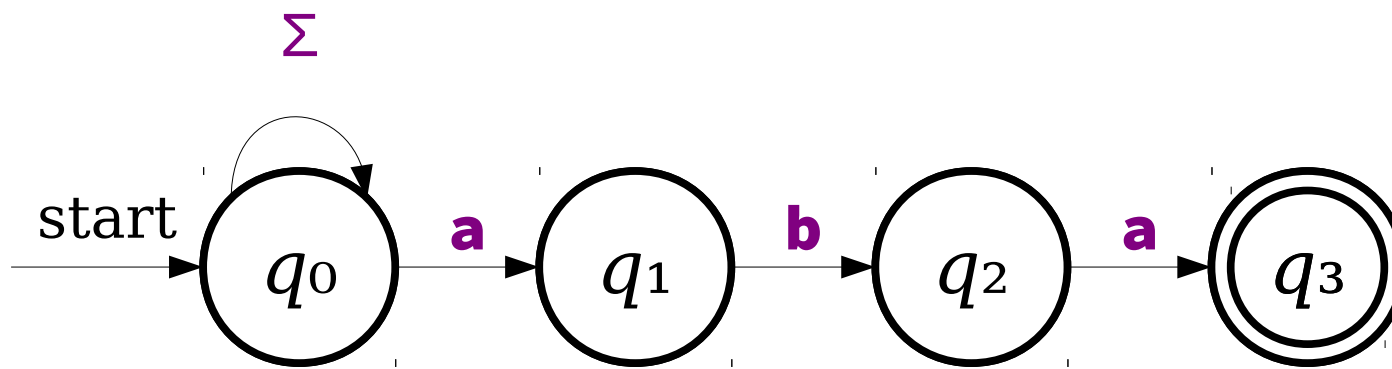




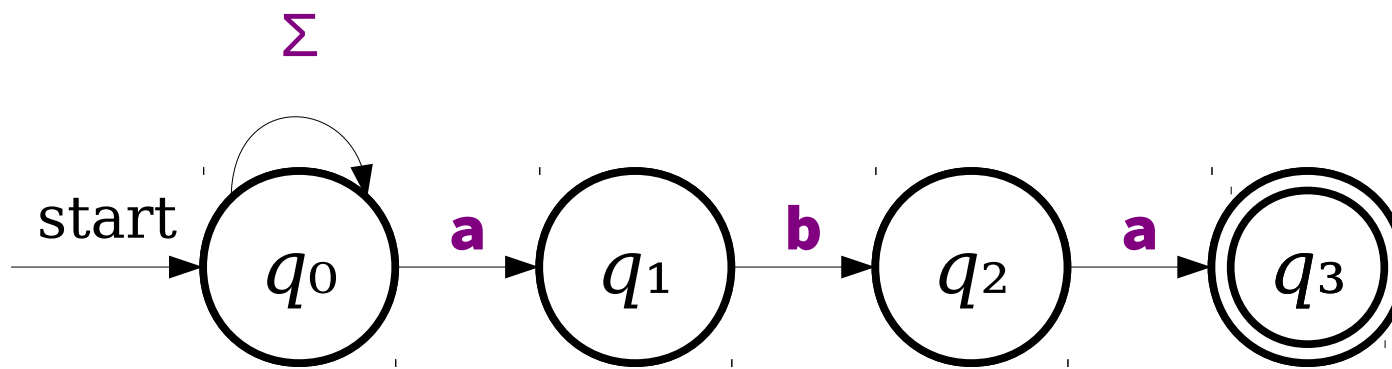




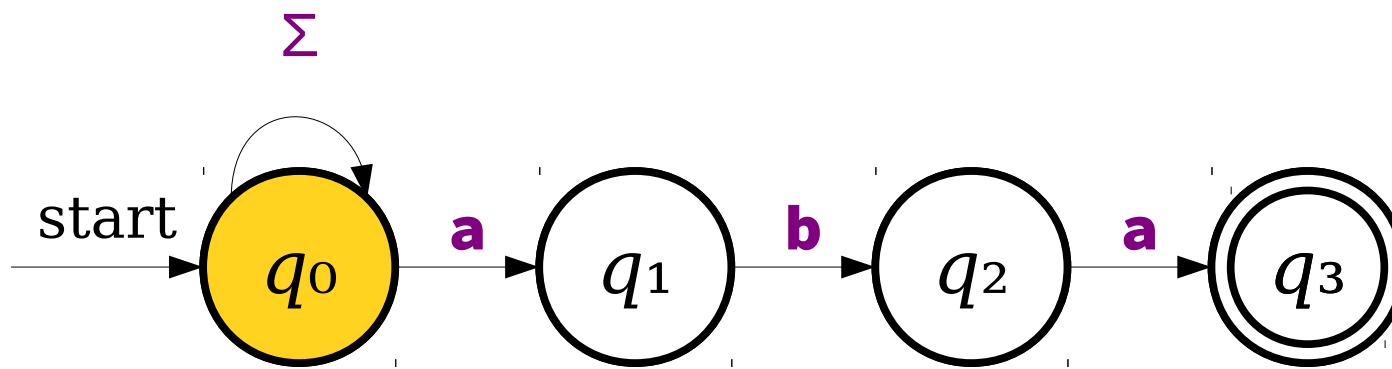




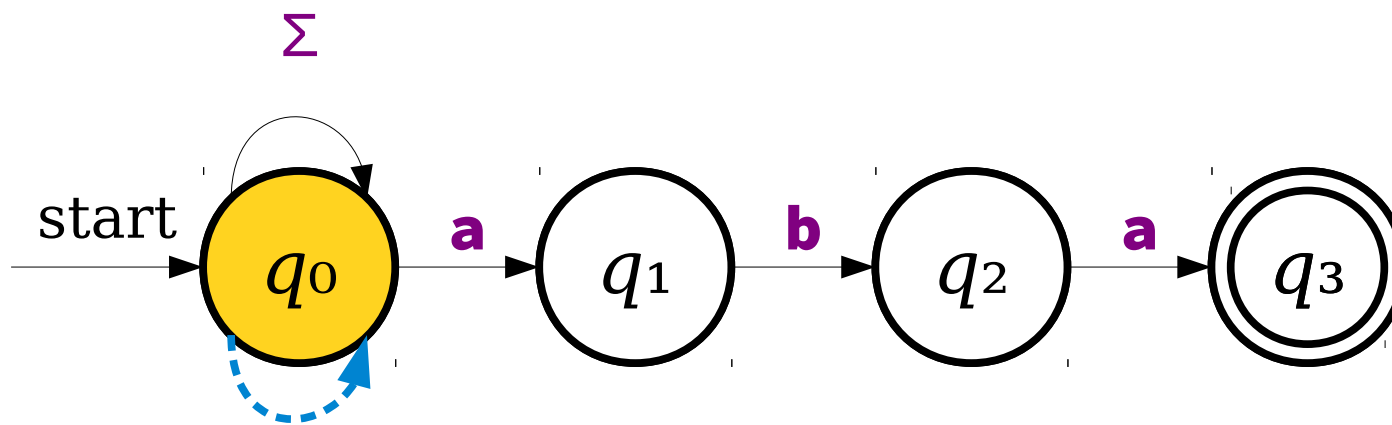
	a
{q ₀ }	{q ₀ , q ₁ }



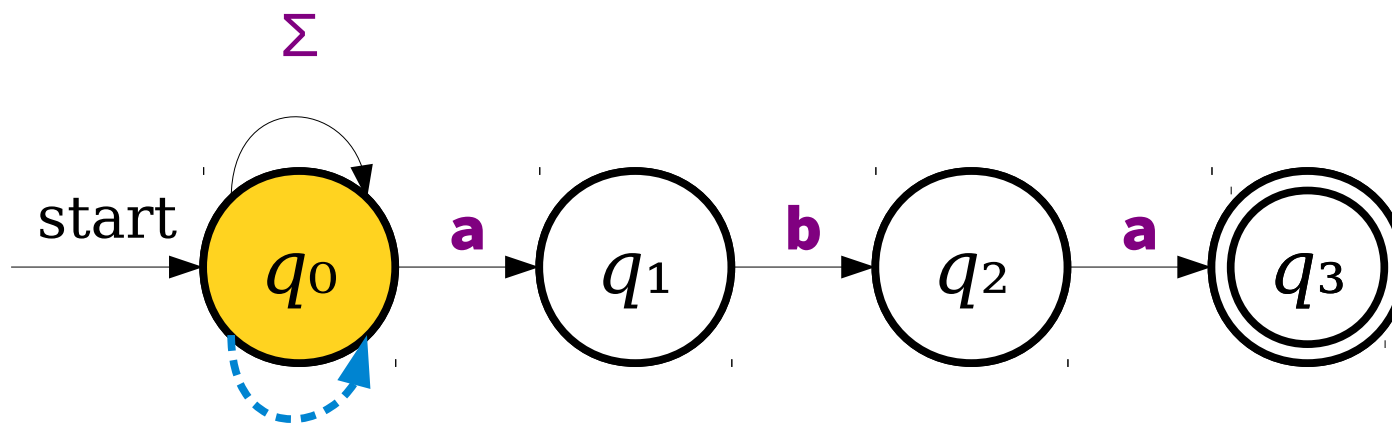
	a	b
{q ₀ }	{q ₀ , q ₁ }	



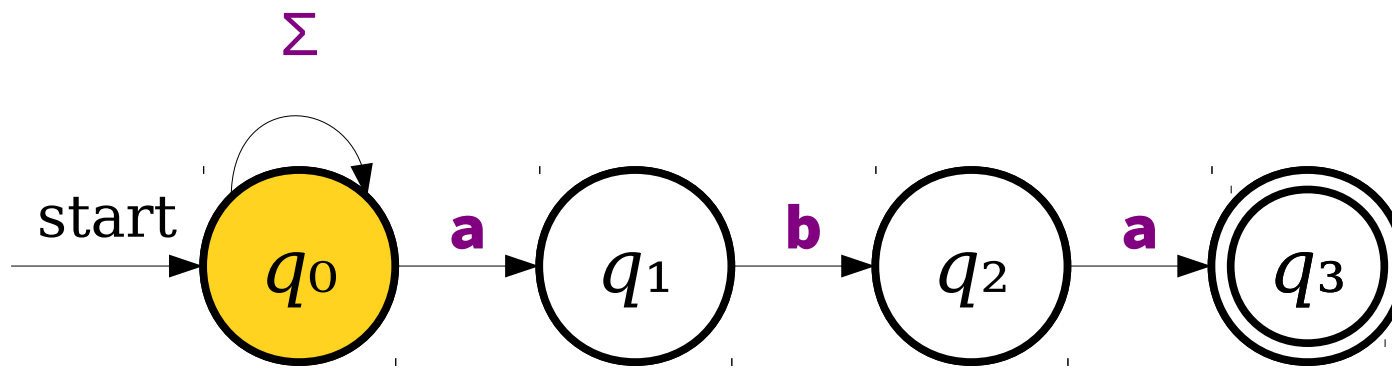
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



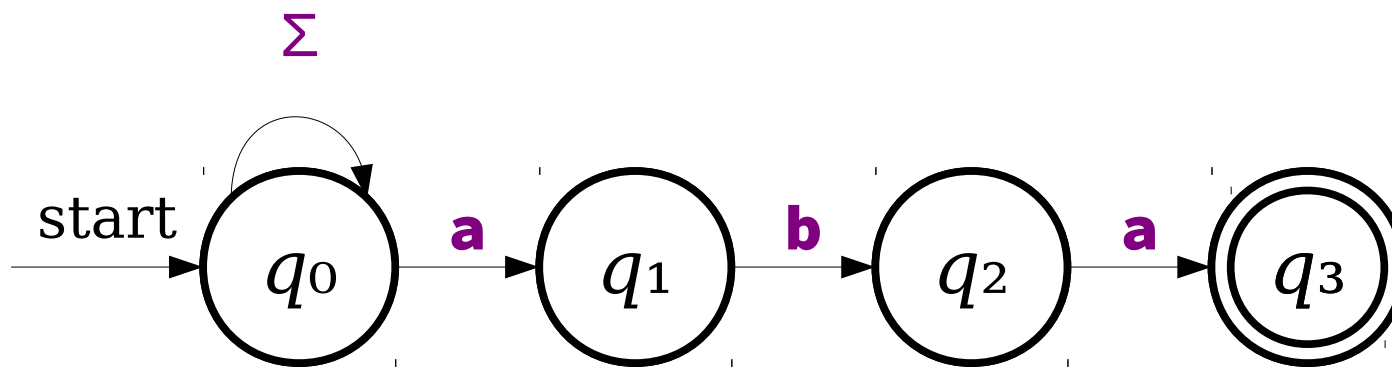
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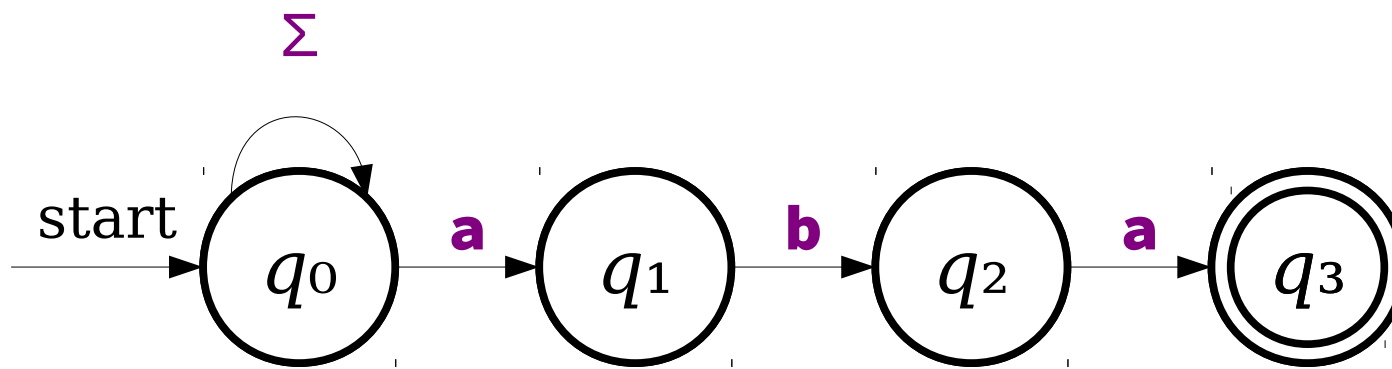
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



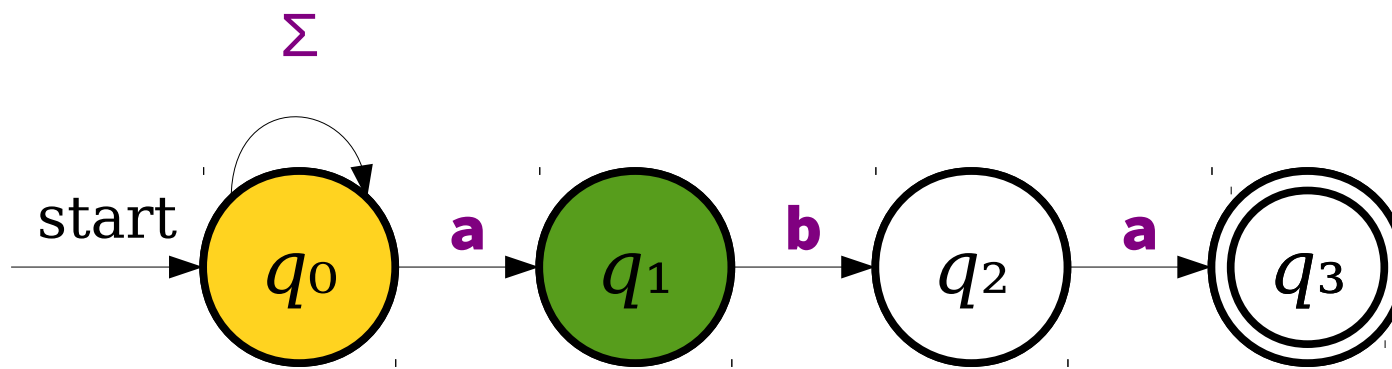
	a	b
{q ₀ }	{q ₀ , q ₁ }	{q ₀ }



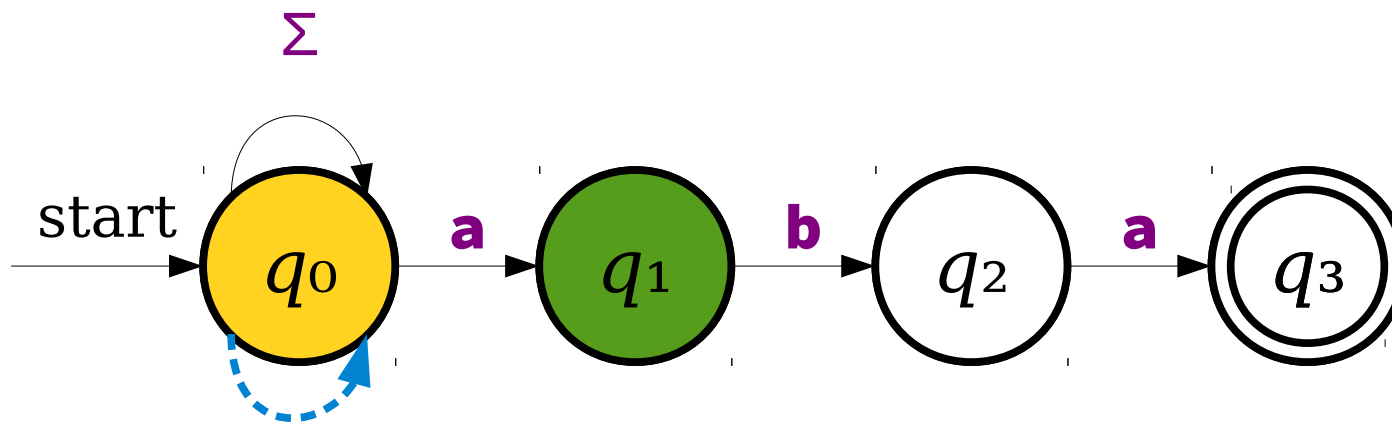
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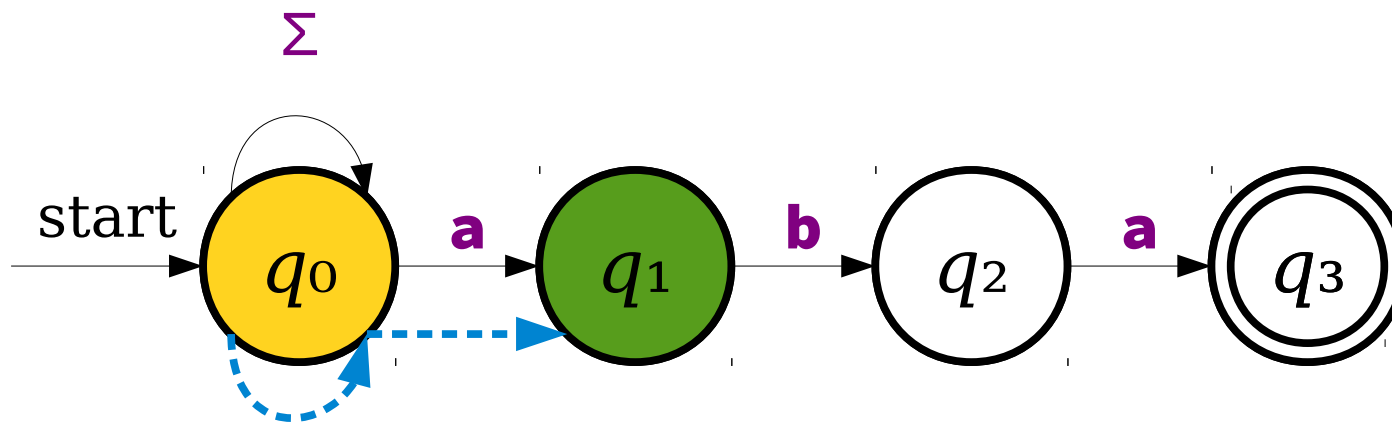
	a	b
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$\{q_0, q_1\}$		



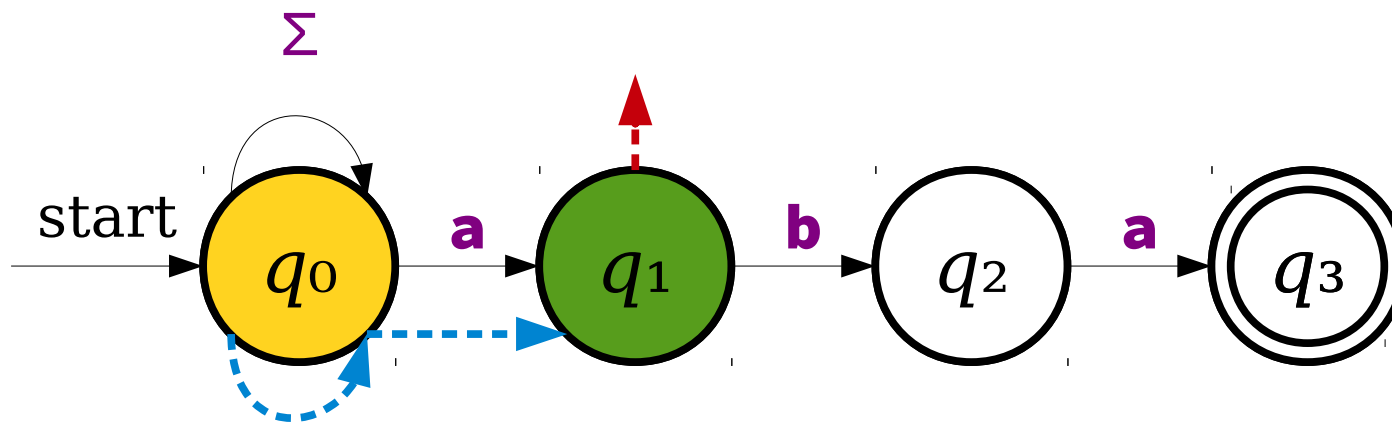
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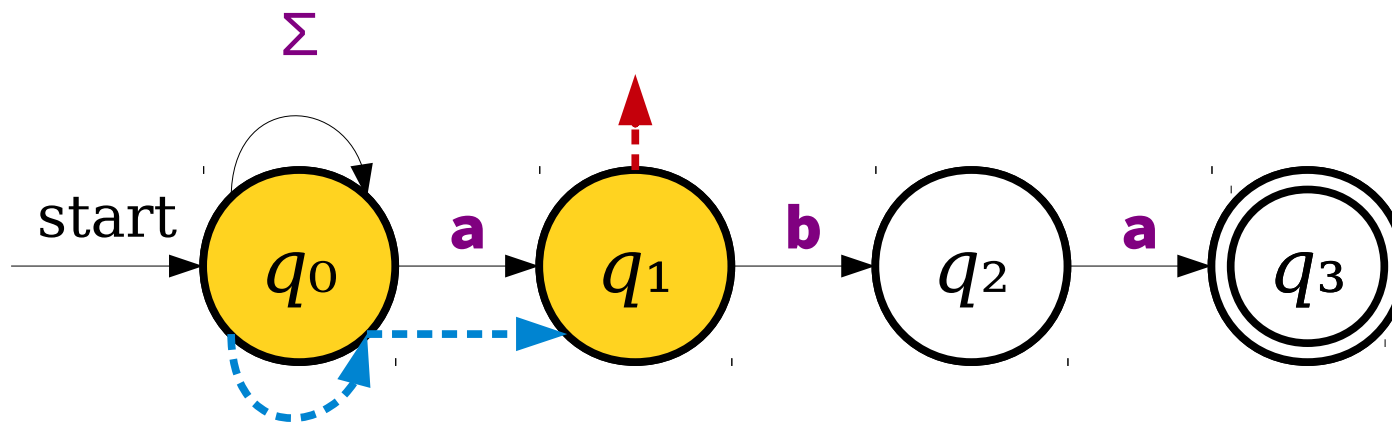
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
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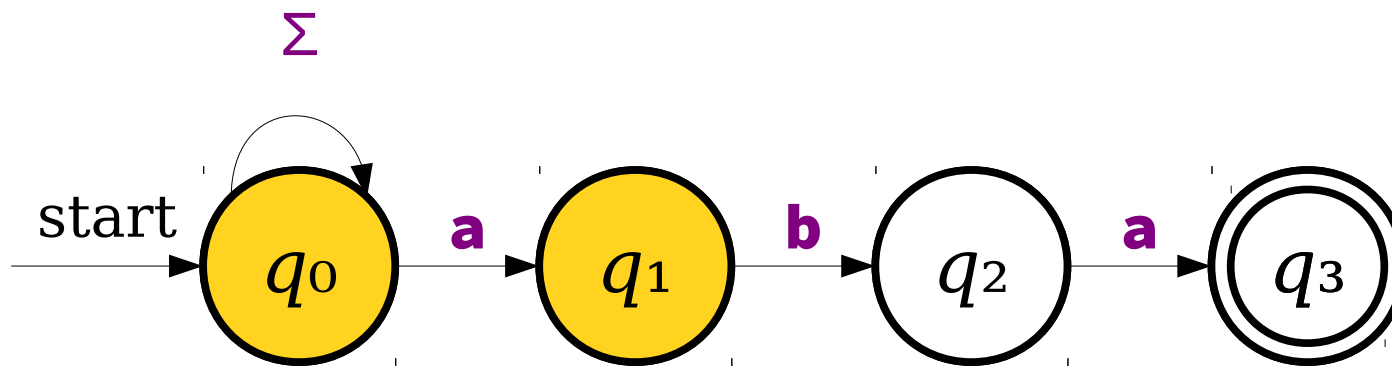
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$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
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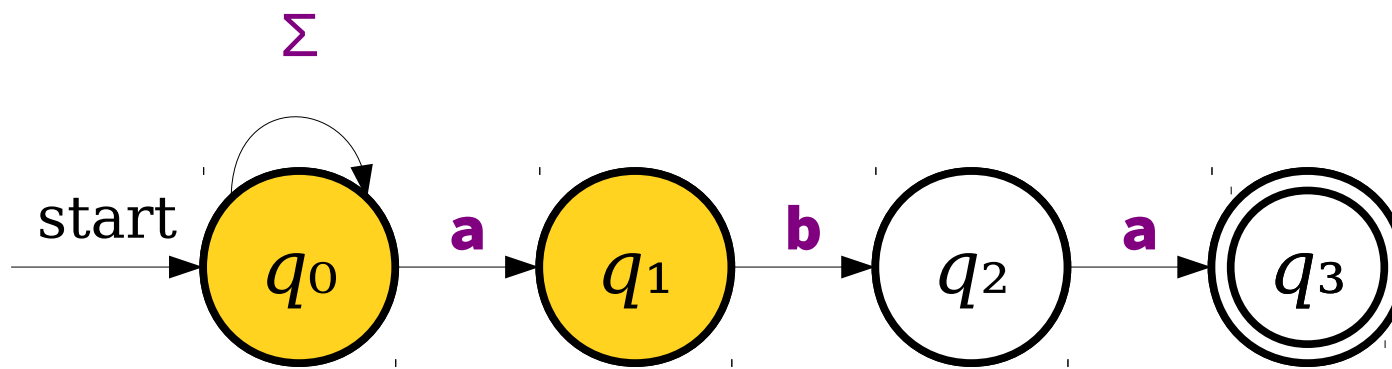
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
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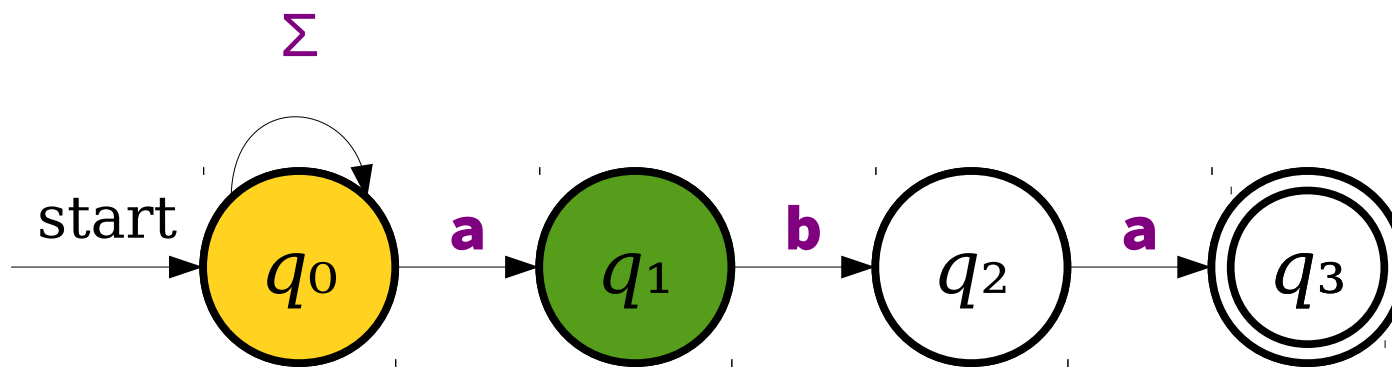
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



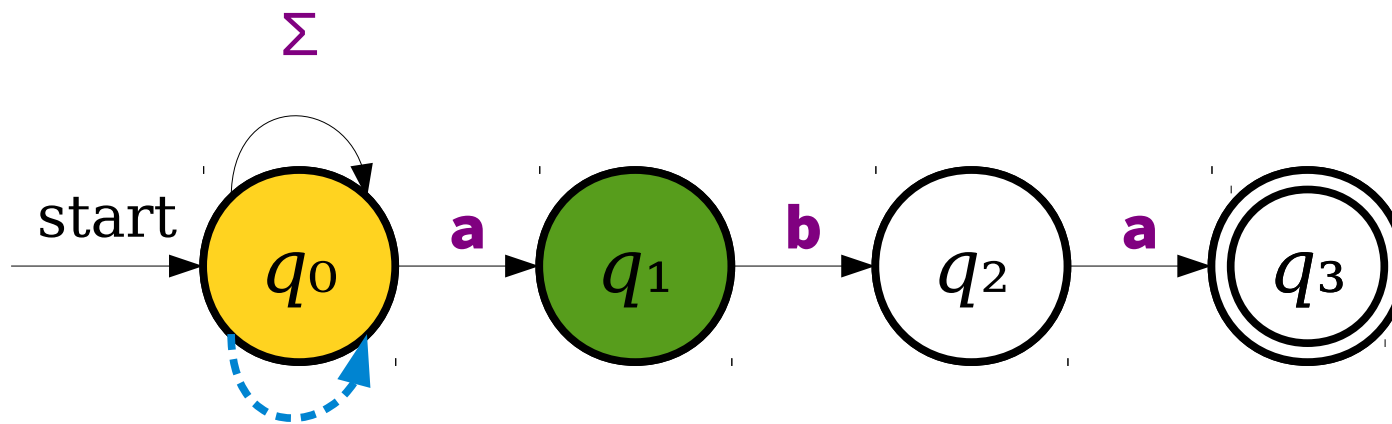
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



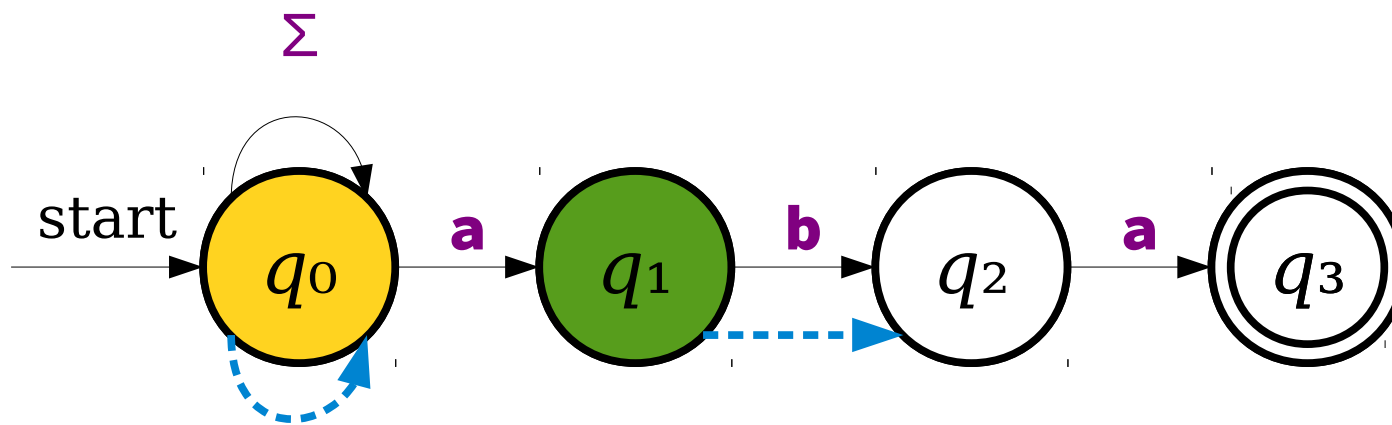
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



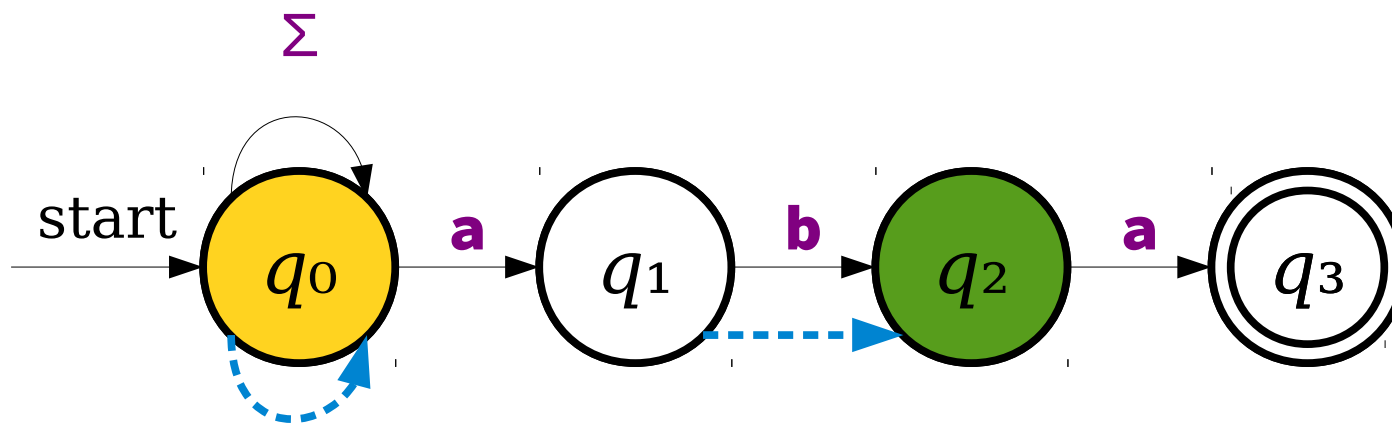
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



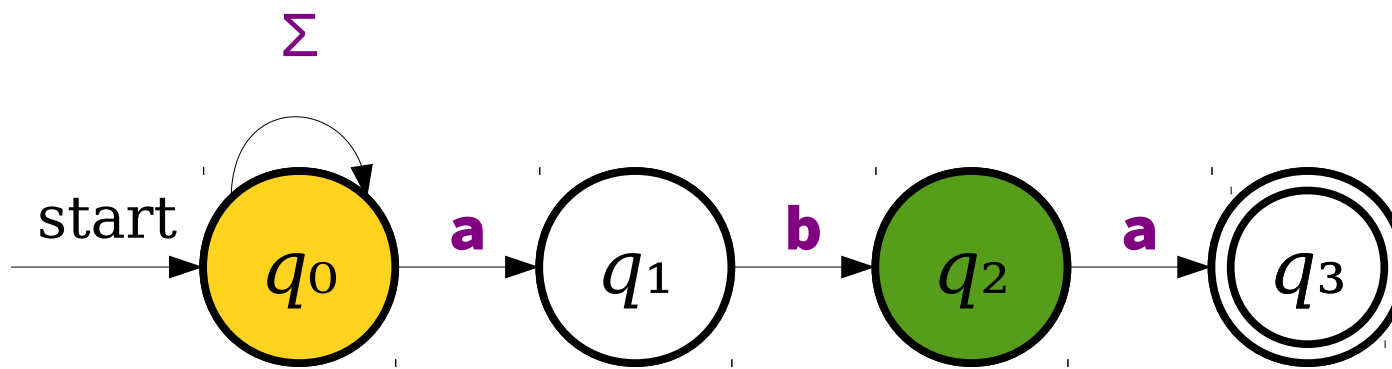
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



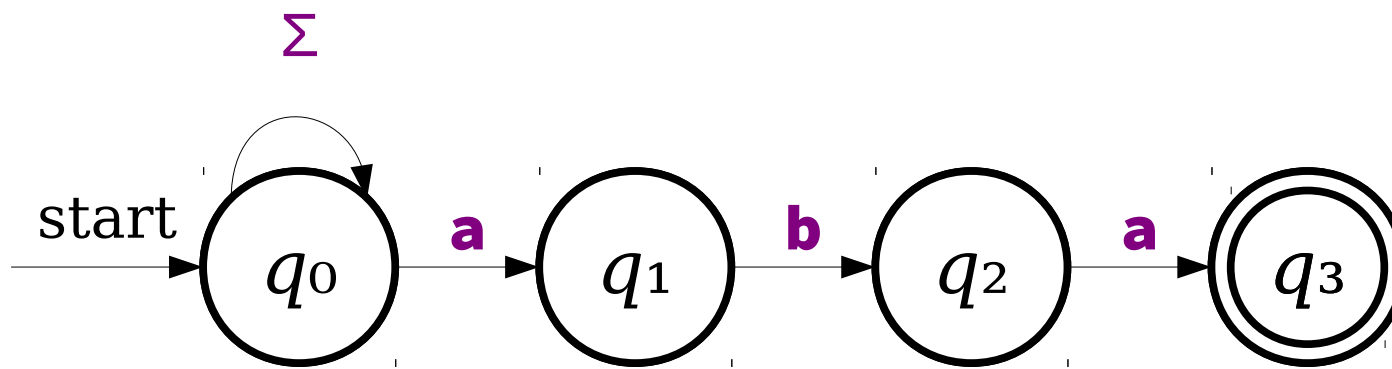
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



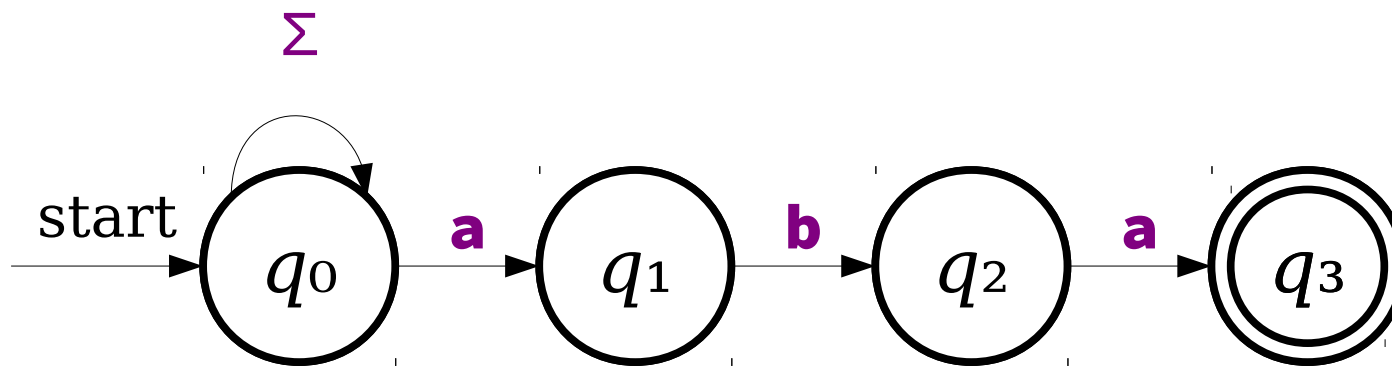
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



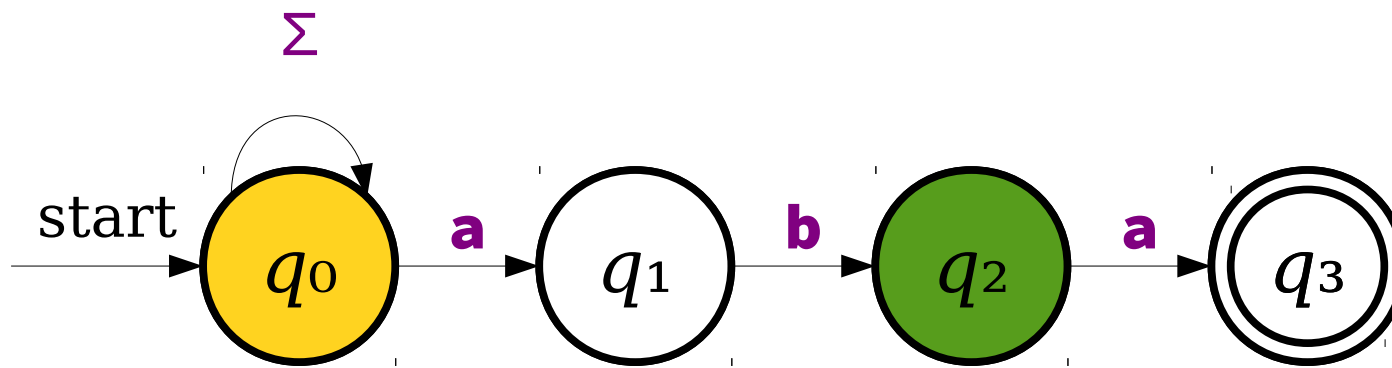
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



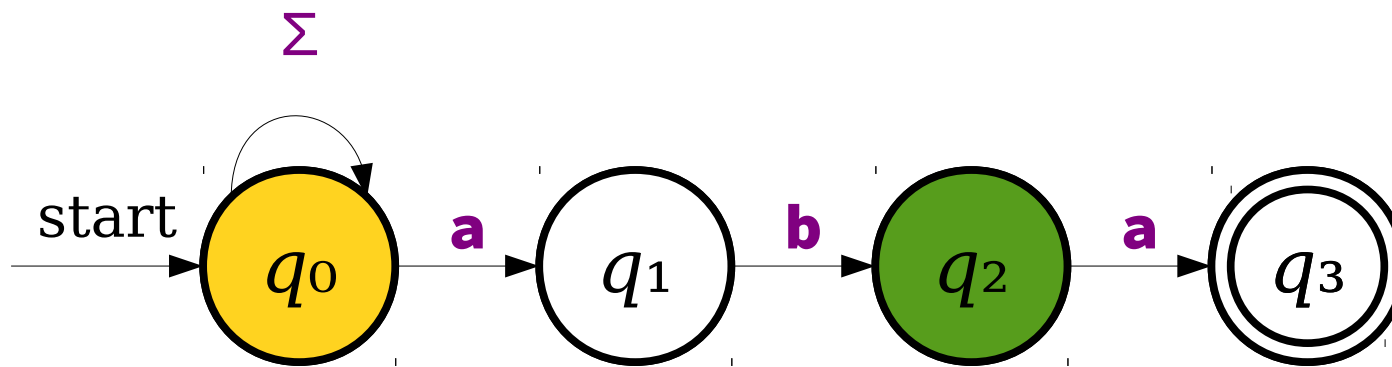
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		

What should this row look like?

- A

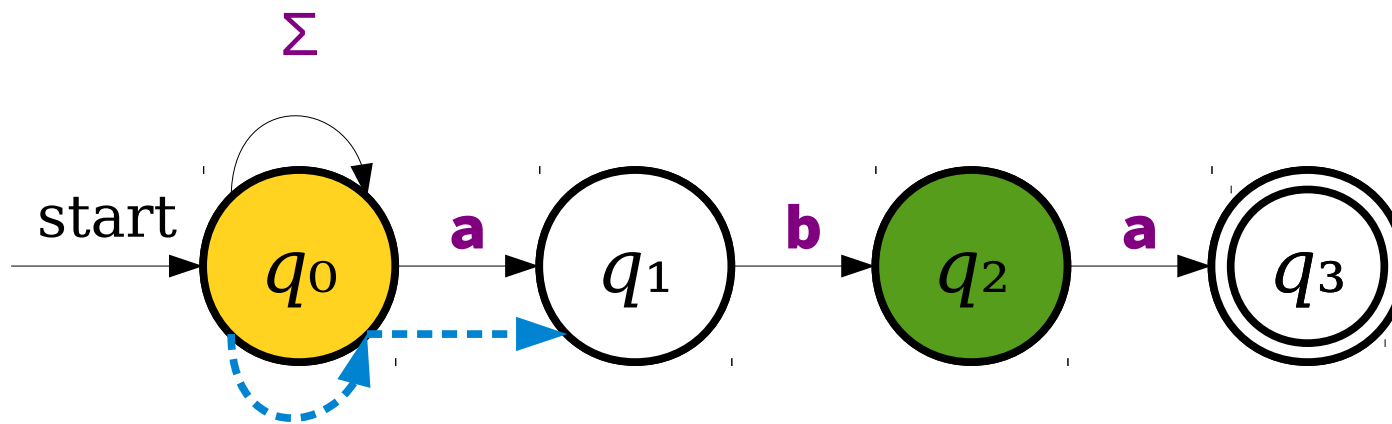
$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0\}$
----------------	----------------	-----------
- B

$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_2\}$
----------------	----------------	----------------
- C

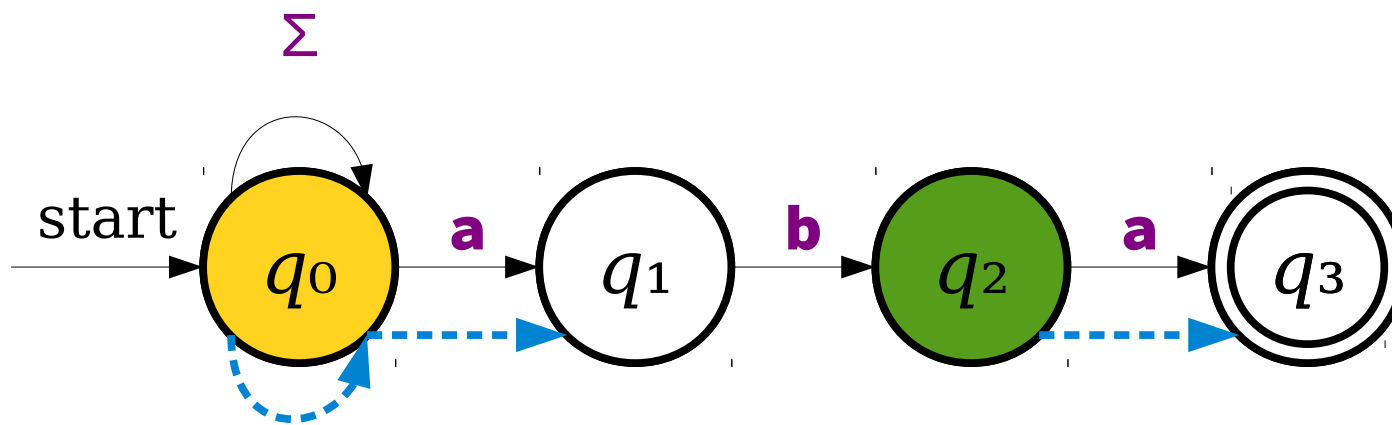
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
----------------	---------------------	-----------
- D

$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$
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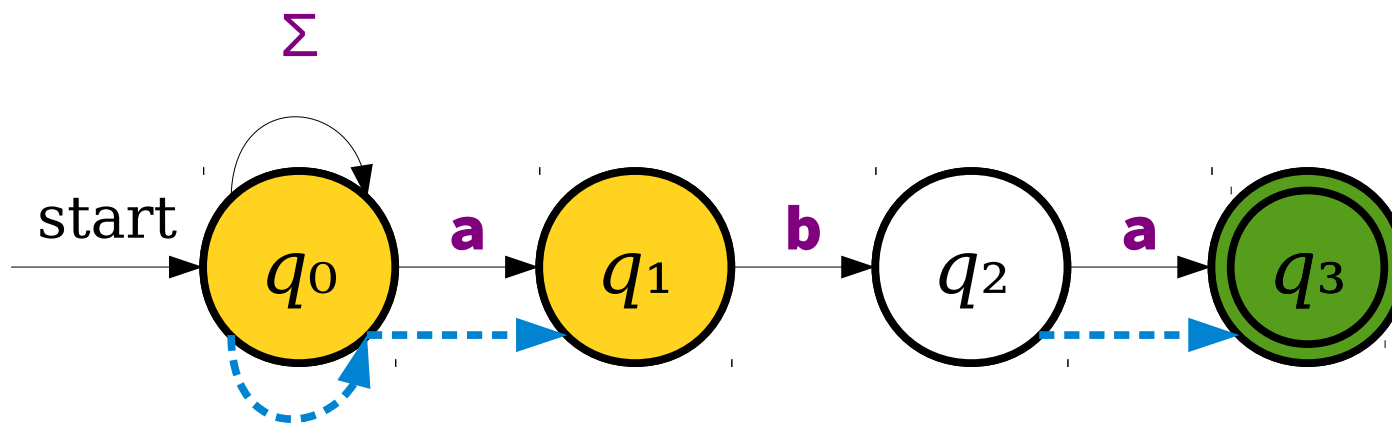
Answer at [PollEv.com/cs103](https://www.pollEv.com/cs103) or
text **CS103** to **22333** once to join, then **A**, **B**, **C**, or **D**.



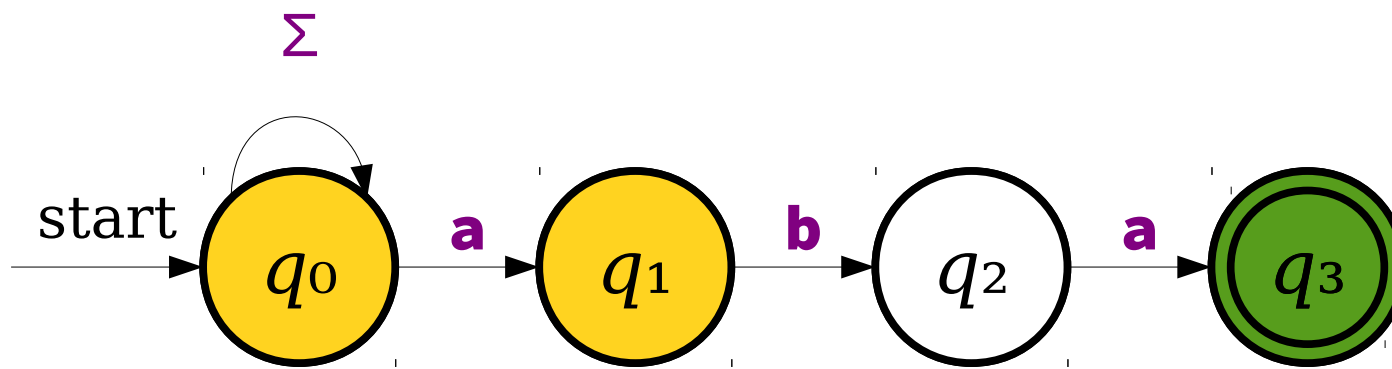
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



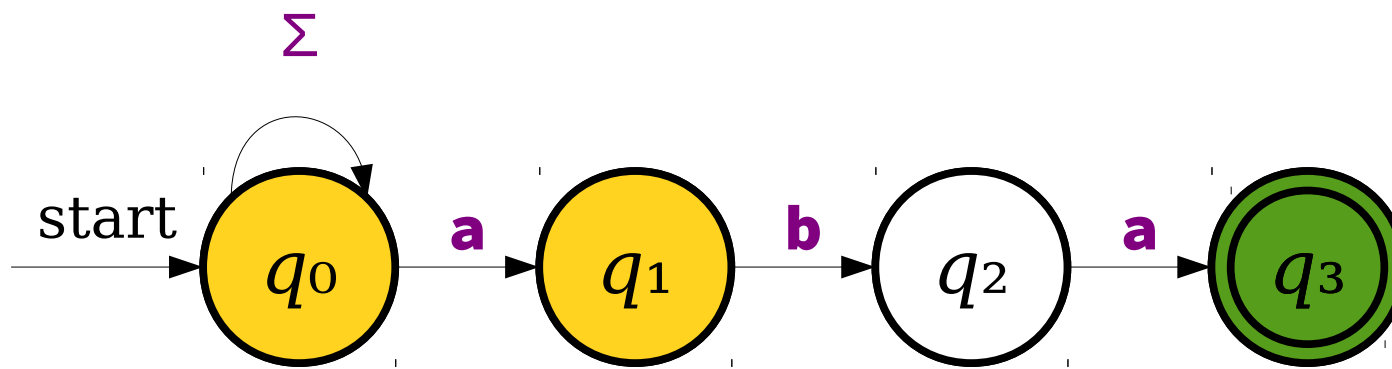
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



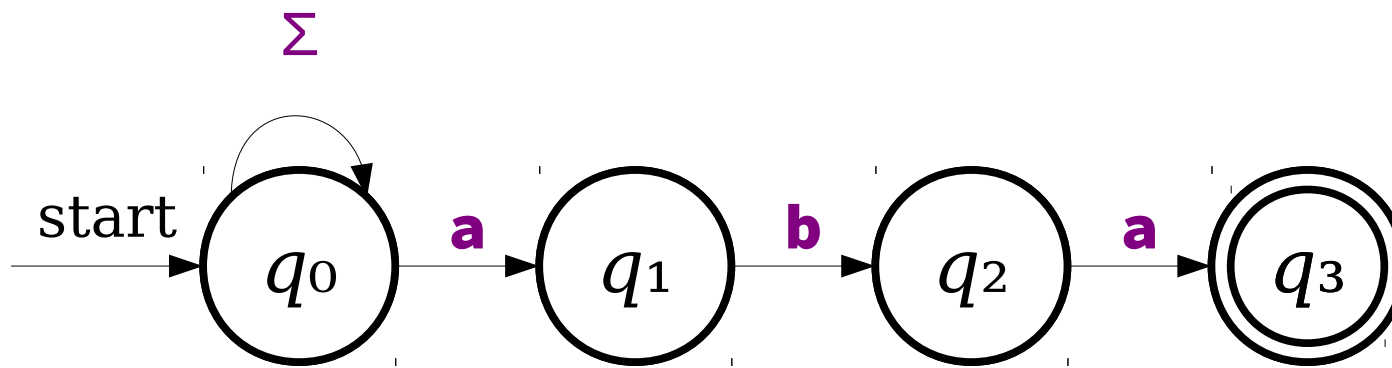
	a	b
{ q_0 }	{ q_0, q_1 }	{ q_0 }
{ q_0, q_1 }	{ q_0, q_1 }	{ q_0, q_2 }
{ q_0, q_2 }		



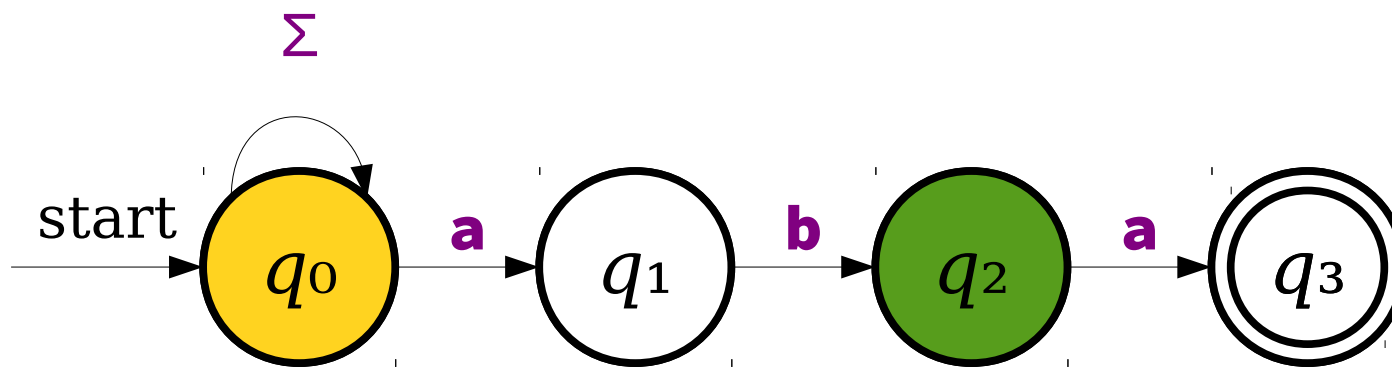
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



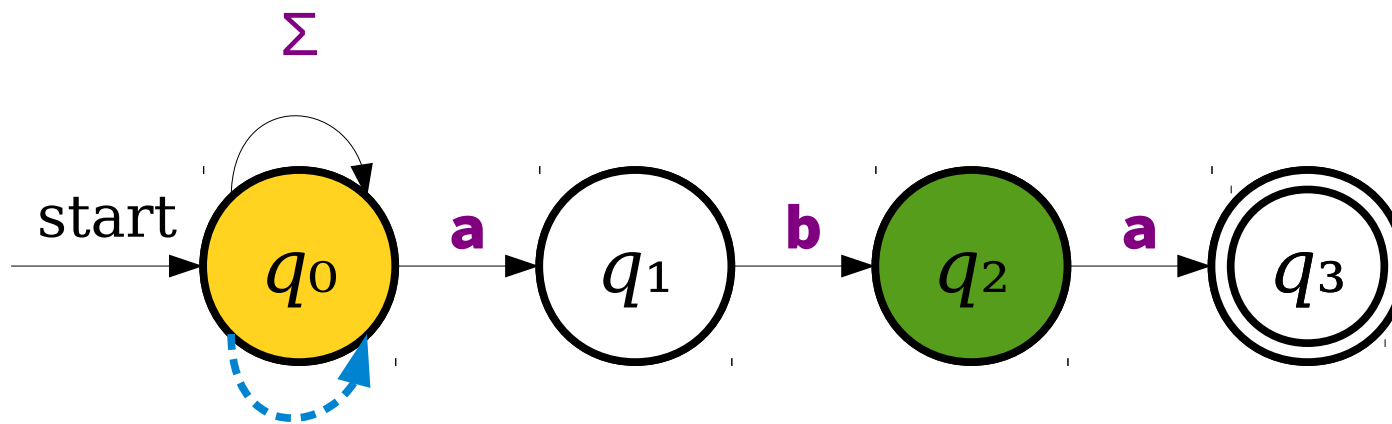
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



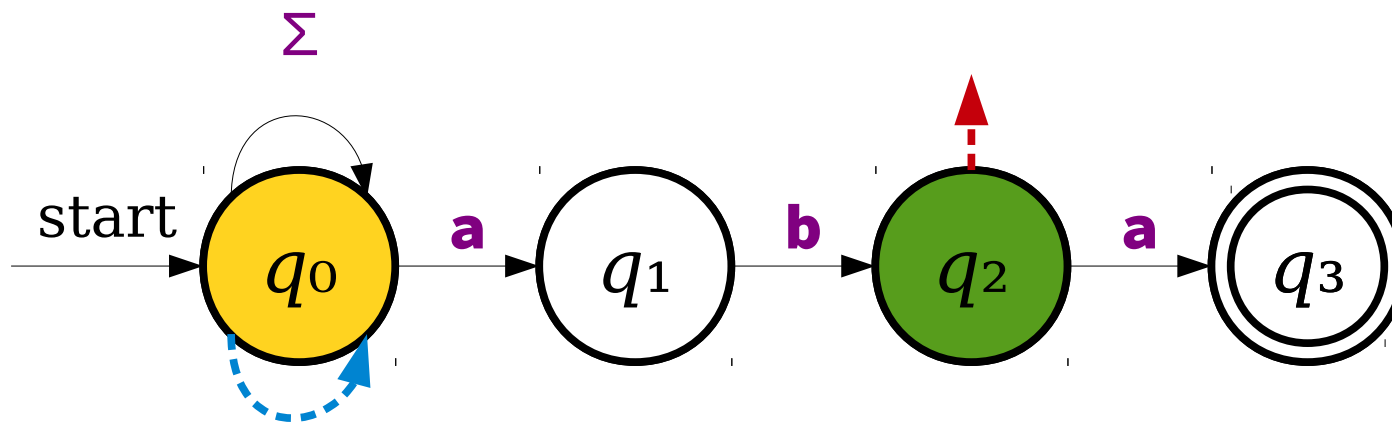
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



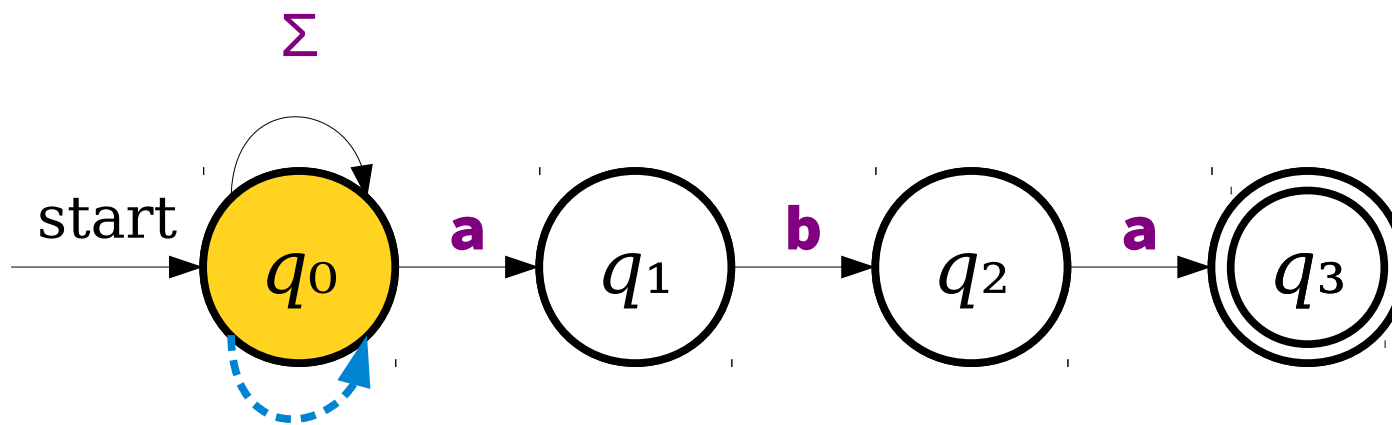
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



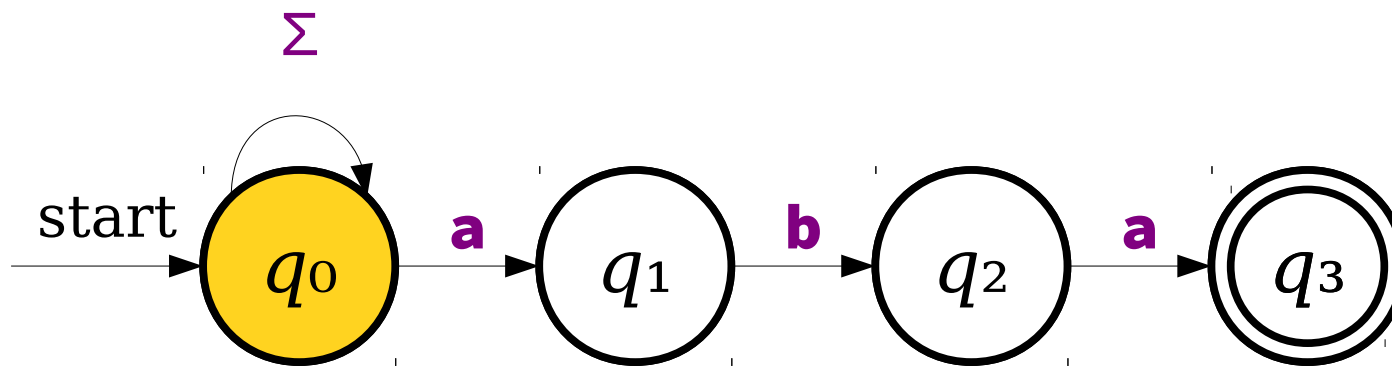
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



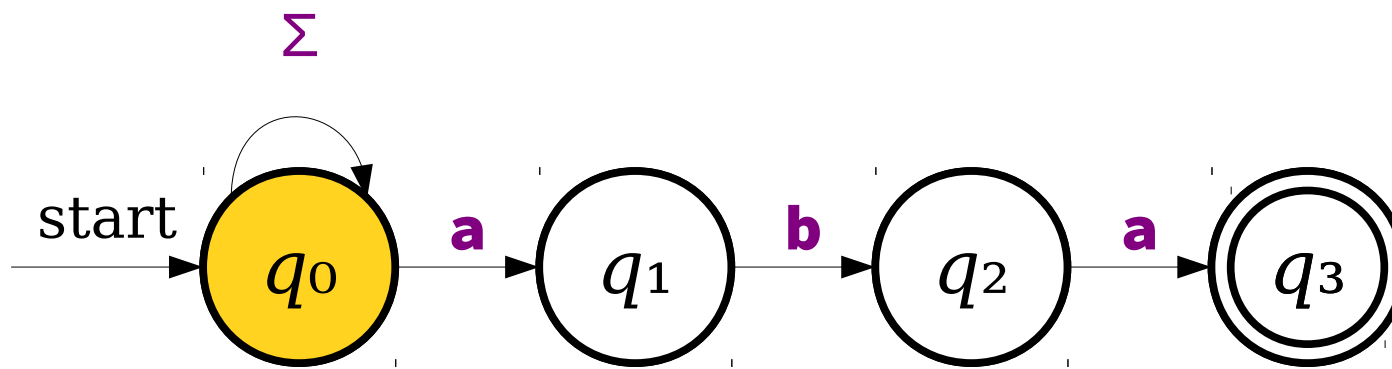
	a	b
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



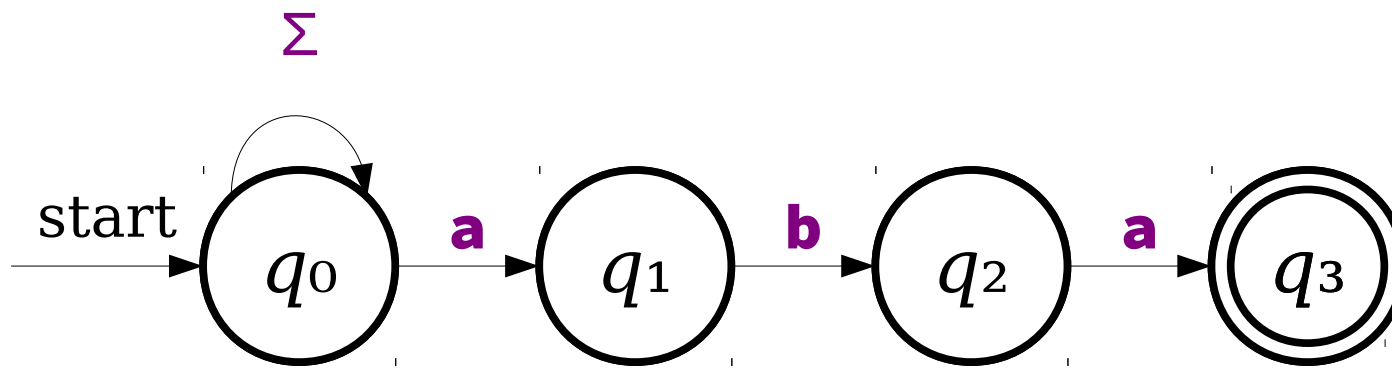
	a	b
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



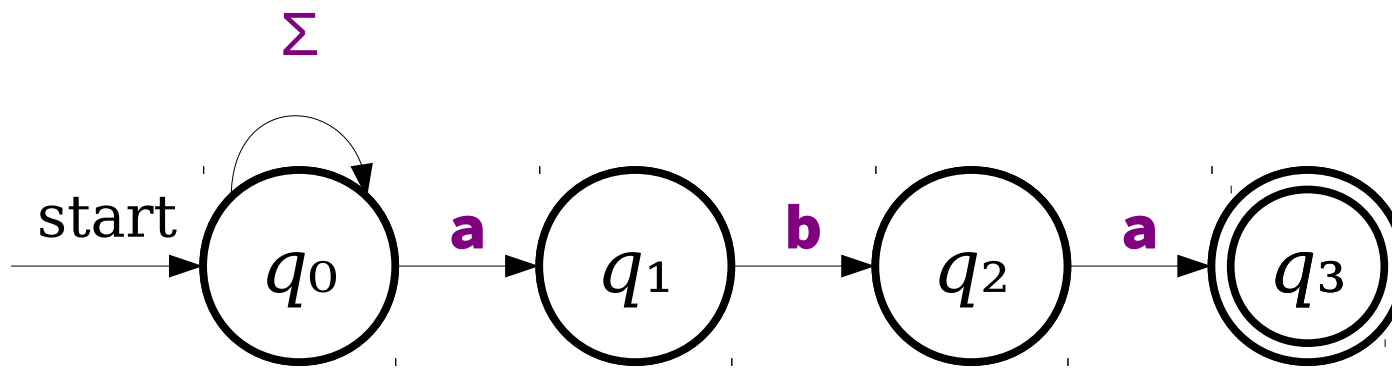
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
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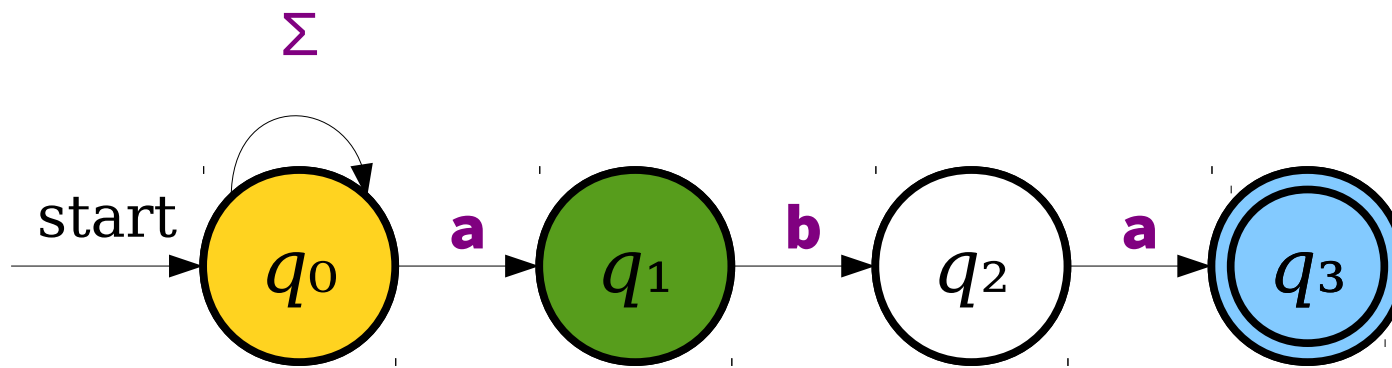
	a	b
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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$



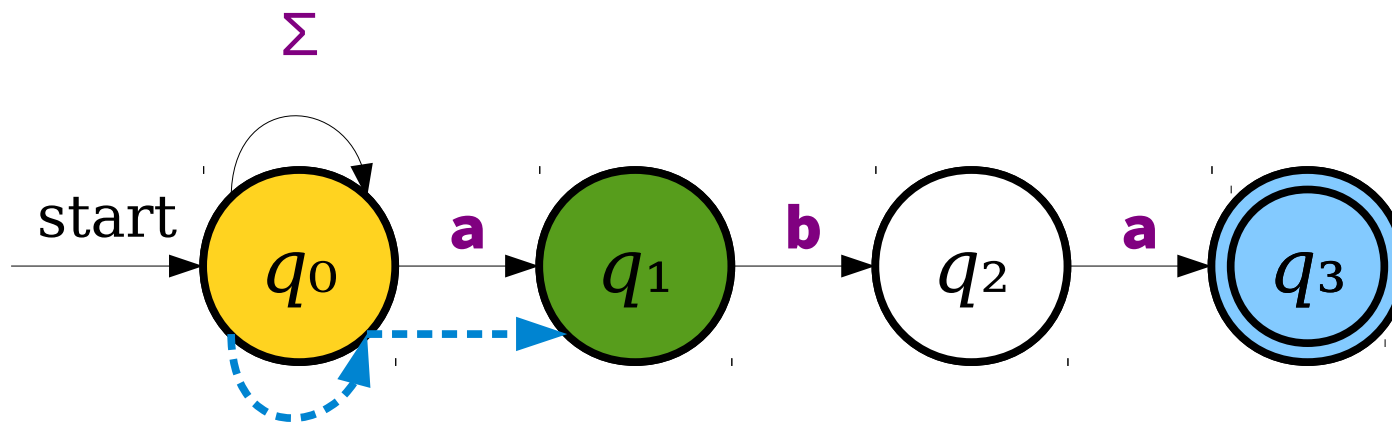
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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$



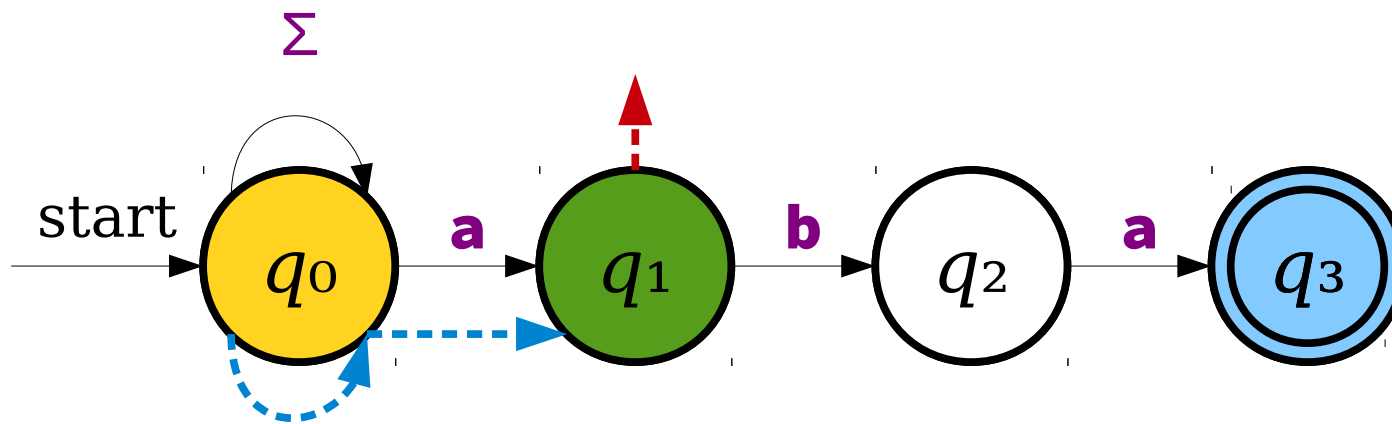
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$\{q_0, q_1, q_3\}$		



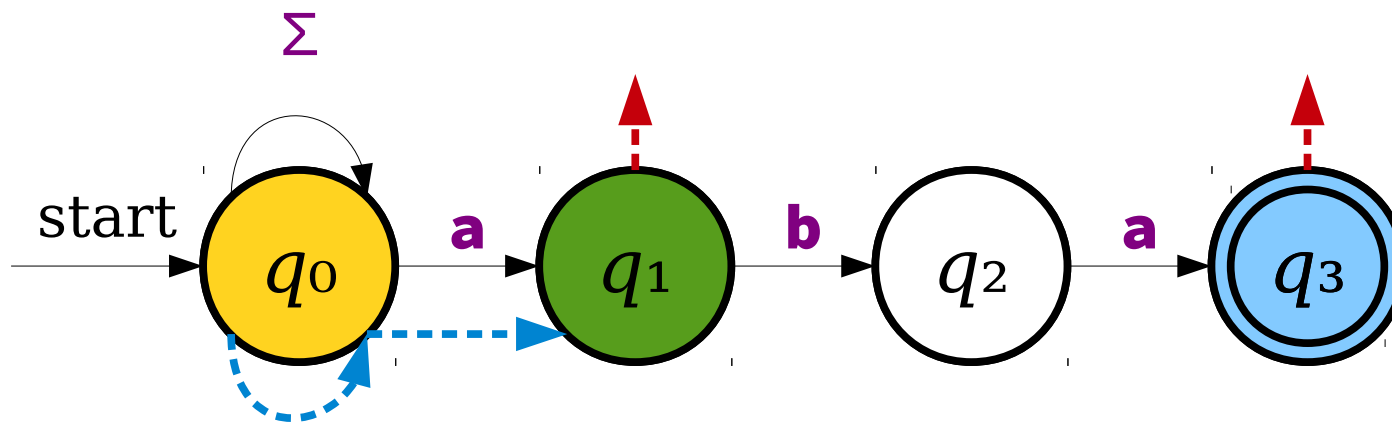
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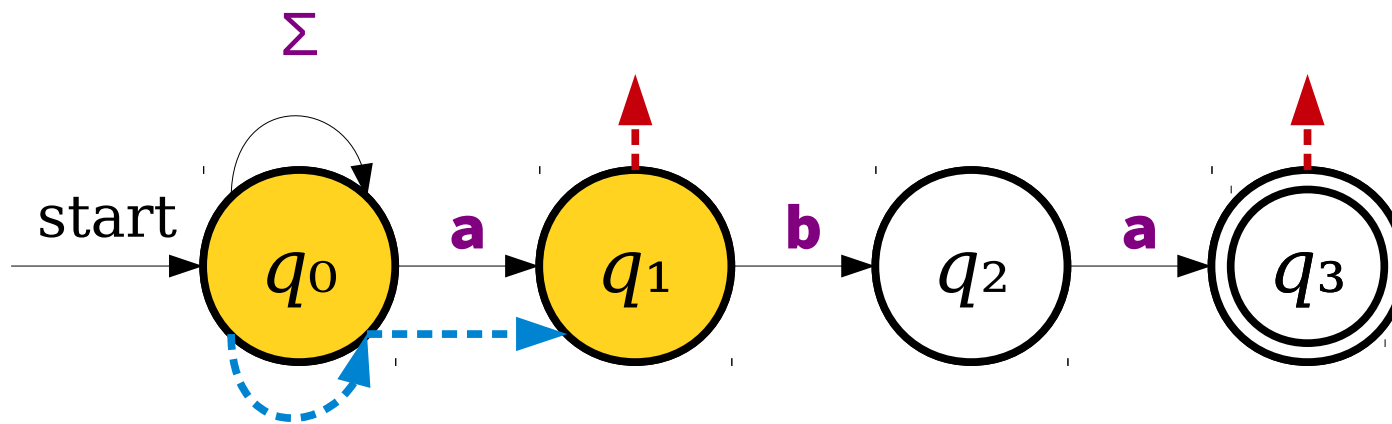
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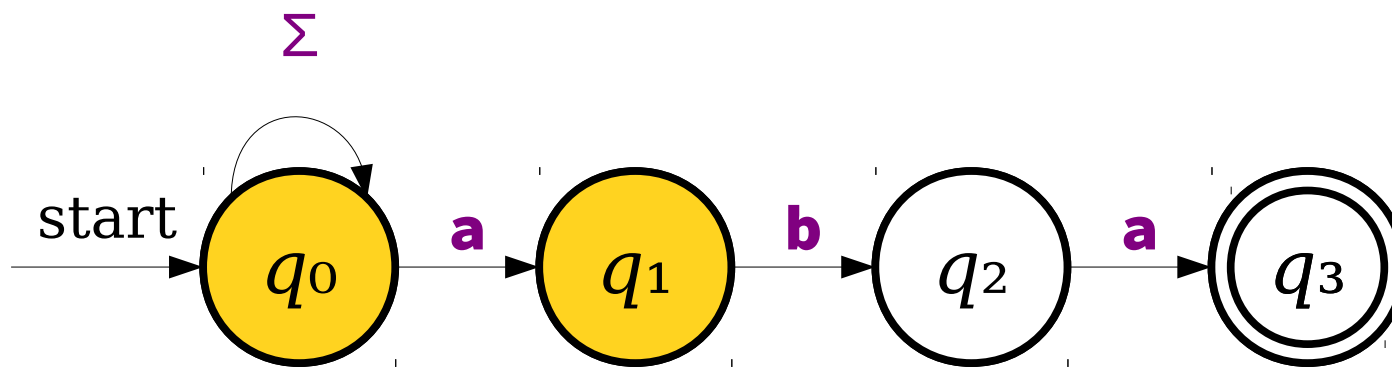
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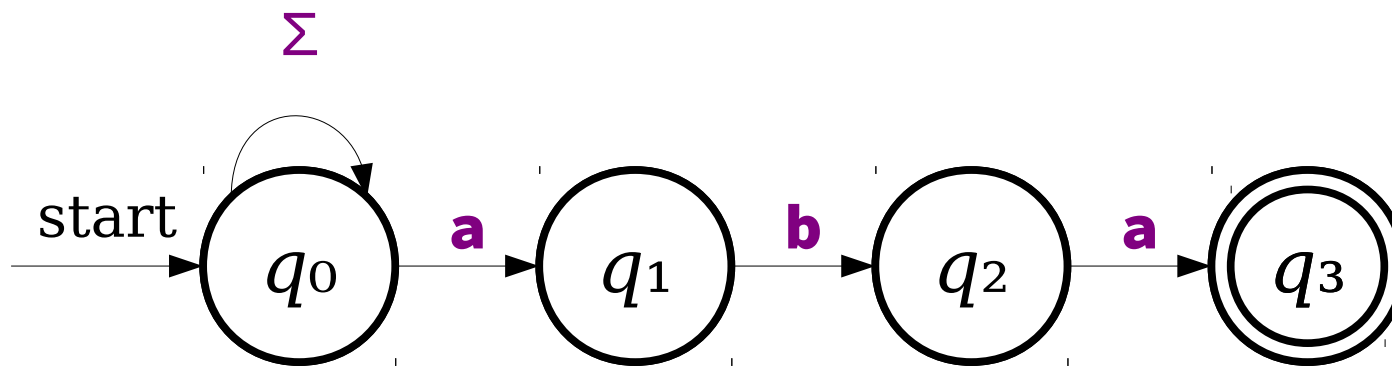
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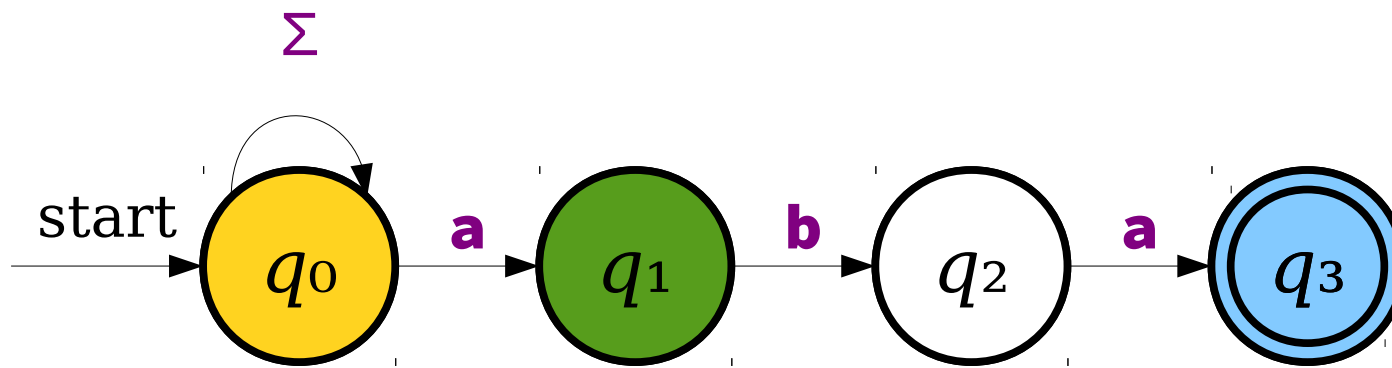
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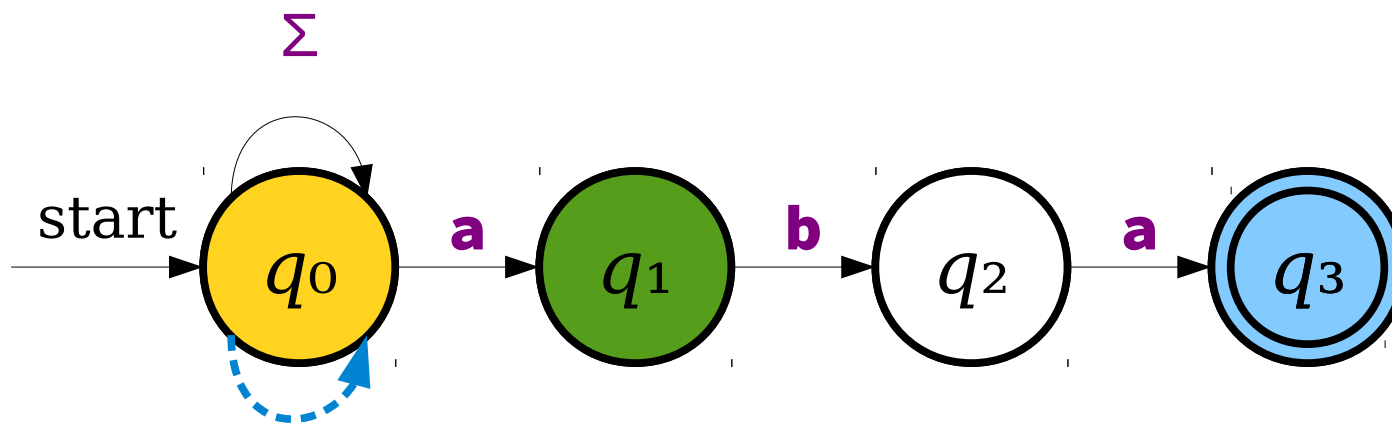
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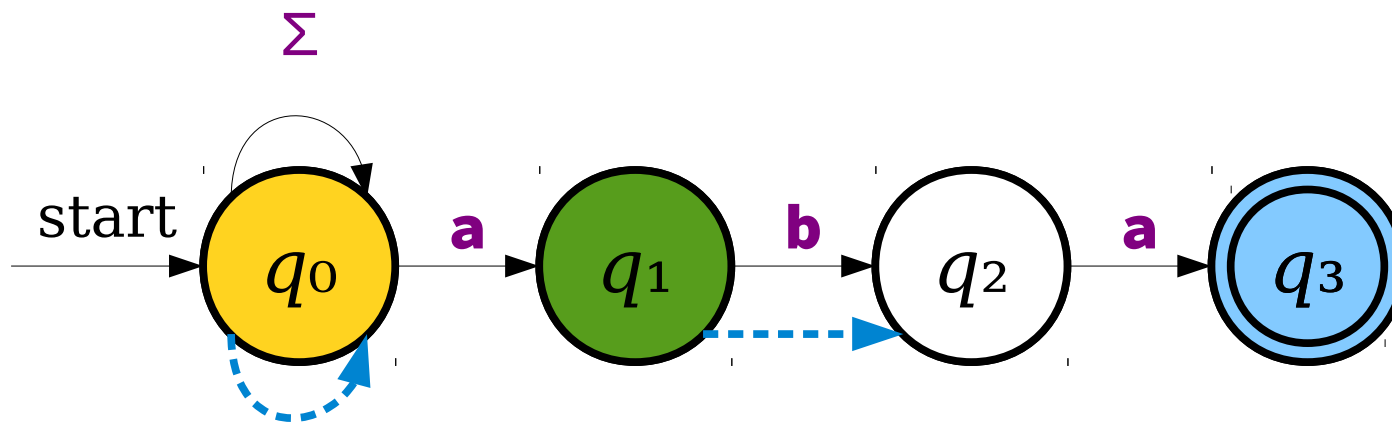
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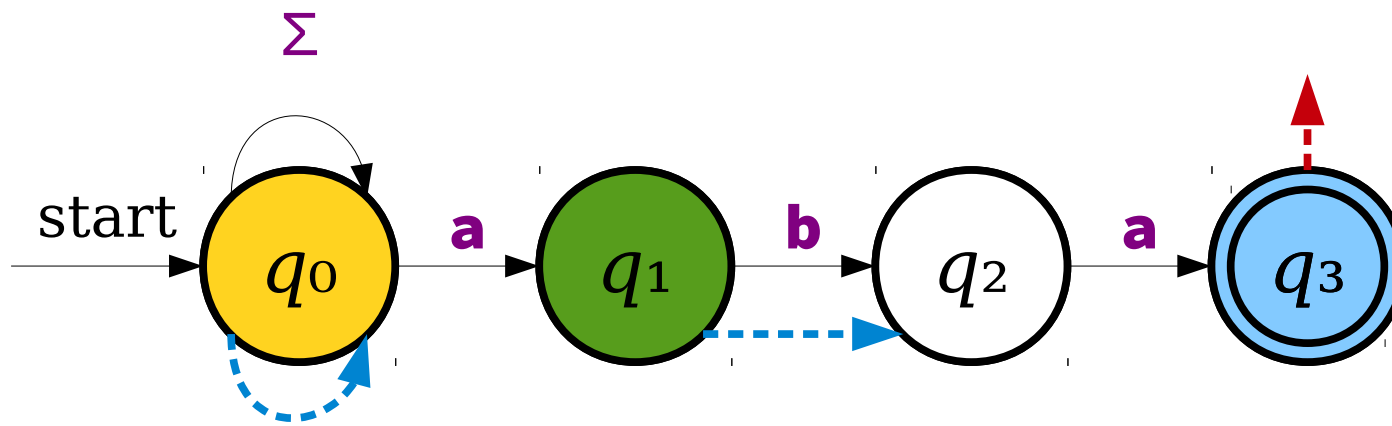
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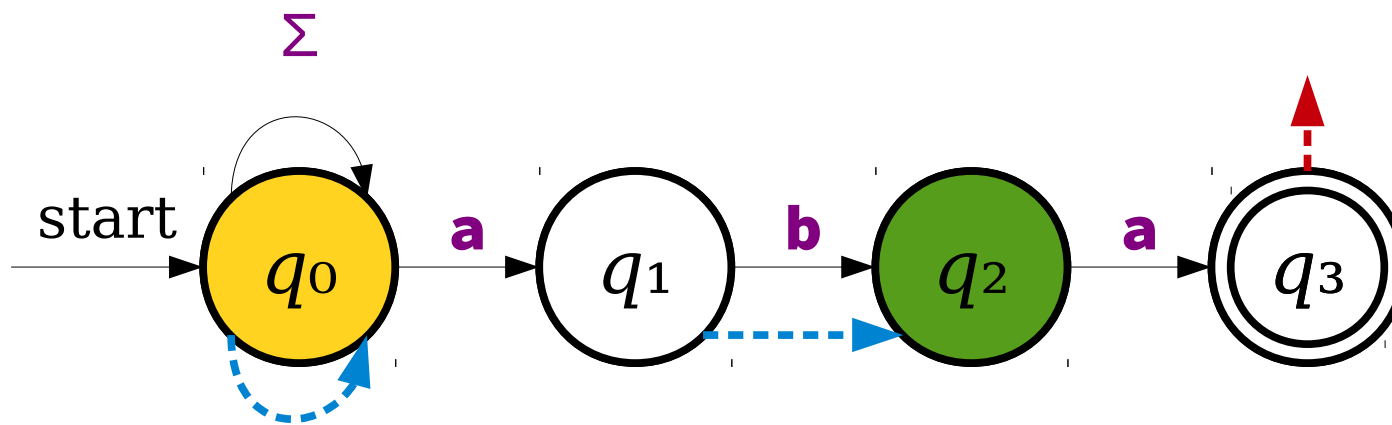
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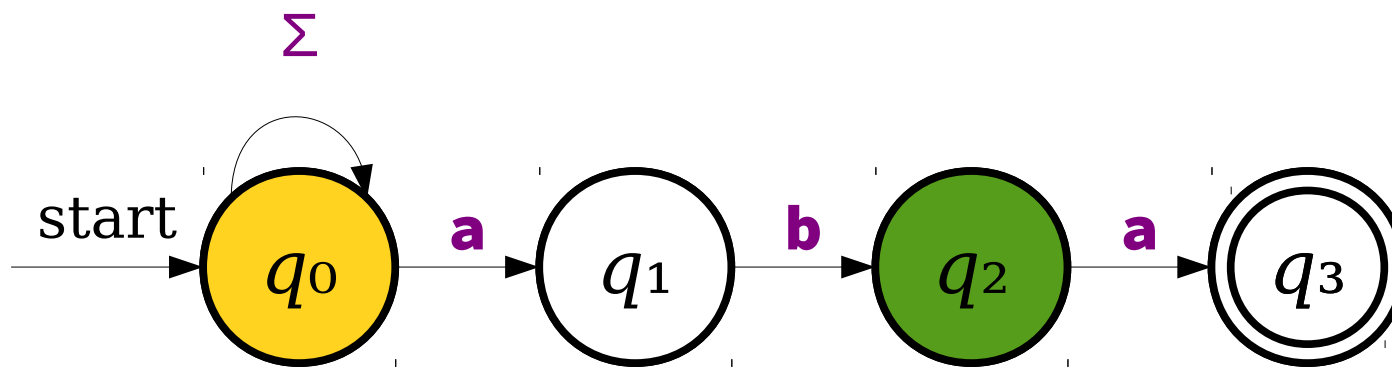
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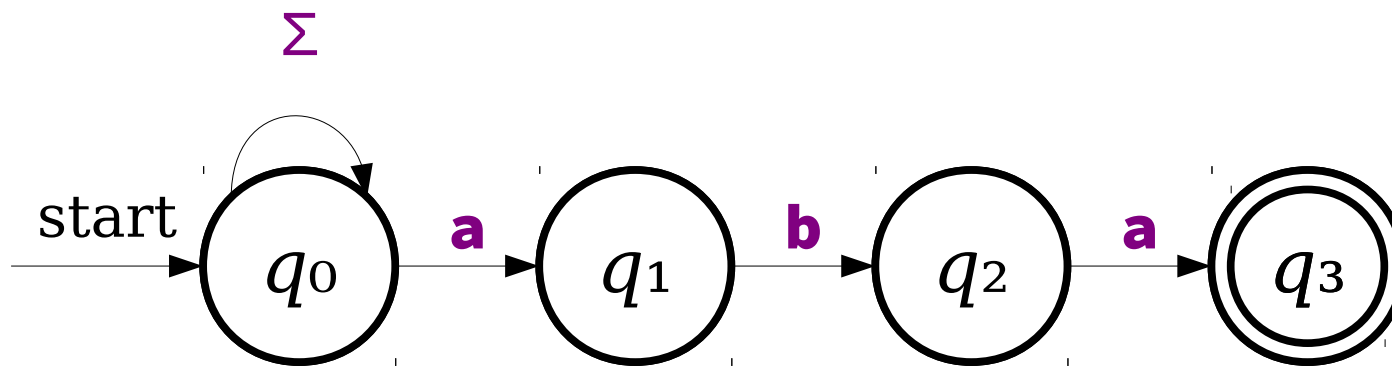
	a	b
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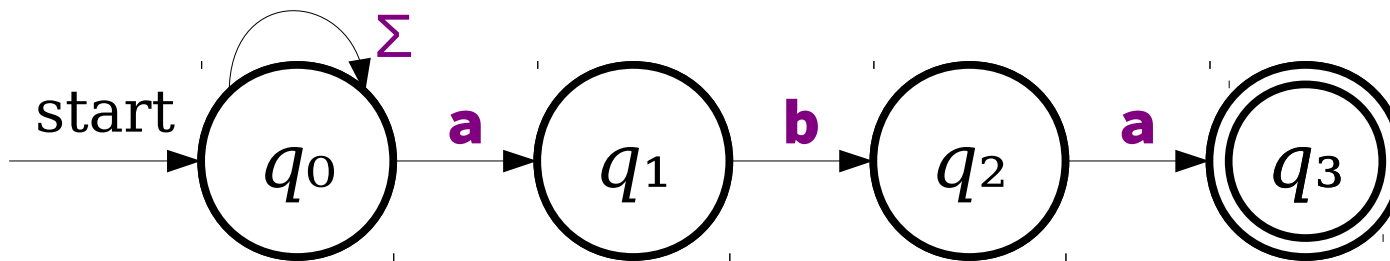
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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
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$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

